Mathe I 2011-06-27

1. " $\iff \log_3(9+3^{x+2}) = 4$

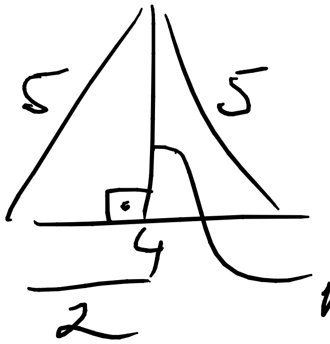
$\iff \underbrace{9+3^{x+2}}_{3^2(1+3^x)} = 3^4$

$\iff 1+3^x = 3^{4-2} = 3^2 = 9$

$\iff 3^x = 9-1 = 8$

$\iff x = \log_3(8)$.

2.



$\sqrt{5^2-2^2} = \sqrt{21}$

$\Rightarrow$ Fläche $= \frac{1}{2} 4 \cdot \sqrt{21} = 2\sqrt{21}$

((ggf auch aufwendiger über Winkelbestimmung))

3. Nenner in Linearfaktoren:

$x^2 - 5x + 6 = 0 \iff x = \frac{5}{2} \pm \sqrt{\underbrace{\frac{25}{4} - 6}_{1/4}}$

$\iff x = 3 \vee x = 2$

Also Nenner $= (x-3)(x-2)$

Zähler in Linearfaktoren:

$$x^3-4x^2+4x = x\underbrace{(x^2-4x+4)}_{\substack{\text{wieder pq-Formel:} \\ (\dots)=0 \Leftrightarrow x = 2\pm\sqrt{4-4} \\ =2}}$$

$$= x(x-2)^2$$

Zusammen: $\text{Bruch} = \frac{x(x-2)^{\cancel{2}}}{(x-3)\cancel{(x-2)}}$

$\Rightarrow$ Nullstellen bei $x=0$ und $x=2$, Polstelle bei $x=3$.

4. Ableitung:

$$x \mapsto 3(\dots)^2 \cdot \frac{1}{e^{2x}}\left(\cos(x)e^x - \sin(x)e^x\right)$$

$$\left(\left(= 3(\sin(x))^2 e^{-3x}(\cos(x)-\sin(x))\right)\right)$$

5. $u=\sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$$\Rightarrow \int_4^9 \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = \int_4^9 2\cos(u)\frac{du}{dx}\,dx$$

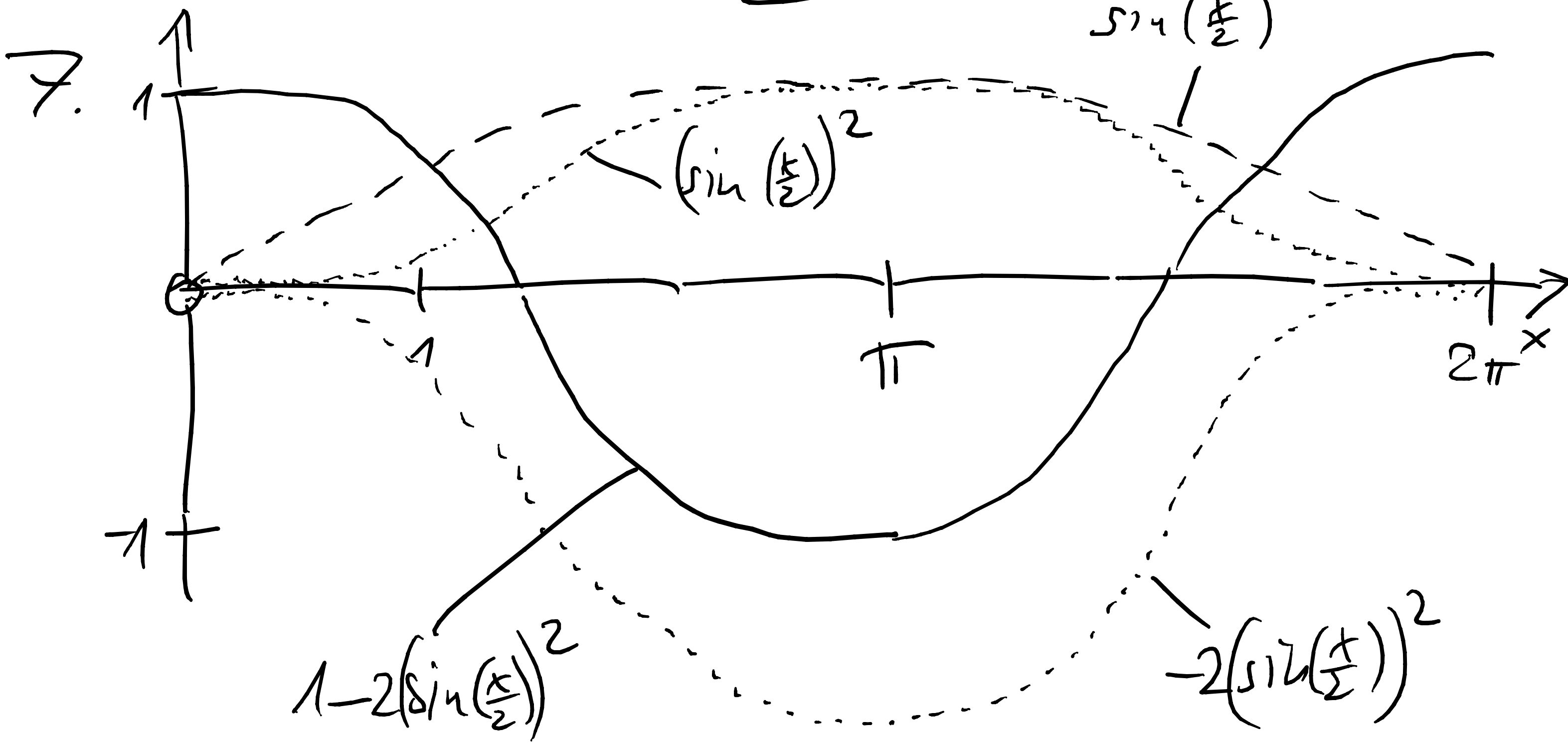
$$\Rightarrow 2\int_2^3 \cos(u)\,du = 2\left[\sin(u)\right]_2^3$$

$$= 2(\sin(3)-\sin(2)).$$

6. $P = \underbrace{P(\{1.K \wedge 2.Z\})}_{\frac{1}{2}\cdot\frac{4}{7}} + \underbrace{P(\{1.Z \wedge 2.K\})}_{\frac{1}{2}\cdot\frac{3}{7}}$

$= \frac{1}{2} \cdot 1 = \frac{1}{2}$

7.



((Ja, das ist einfach $\cos(x)$!))

8. $z^5 = z^2 \iff \underbrace{z^5 - z^2}_{z^2(z^3-1)} = 0$

$\iff z = 0 \vee z^3 = 1$

$\iff z = 0 + 0i$

$\vee\ z = 1 + 0i$

$\vee\ z = \cos(120°) + i\sin(120°) \quad \left(\left(= -\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right)$

$\vee\ z = \cos(120°) - i\sin(120°)$

9. $|x-1| \leq 3x+4$

$\Leftrightarrow$ ⁒ $\wedge x-1 \geq 0$

$\vee$ ⁒ $\wedge x-1 < 0$

$\Leftrightarrow x-1 \leq 3x+4 \wedge x \geq 1$

$\vee 1-x \leq 3x+4 \wedge x < 1$

$\Leftrightarrow -5 \leq 2x \wedge x \geq 1$

$\vee -3 \leq 4x \wedge x < 1$

$\Leftrightarrow \cancel{x \geq -5/2} \wedge x \geq 1$

$\vee x \geq -3/4 \wedge x < 1$

Also $\mathbb{L} = [1;\infty) \cup [-3/4;1)$

$= [-3/4;\infty)$.

10. $\frac{d(x^3+2x+5)}{dt} = 3x^2+2$

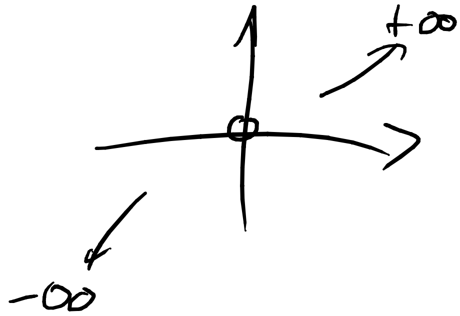
Ist immer > 0!

$\Rightarrow$ Funktion streng monoton wachsend, kann nicht zum Wert 0 zurückkehren.

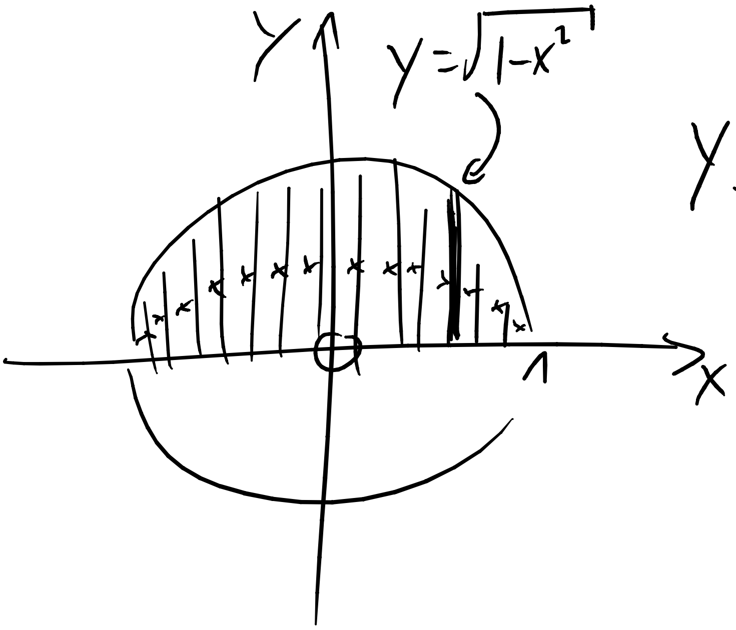
Muss aber auf jeden Fall einmal den Wert 0 annehmen,

$\Downarrow$

weil führender Term x^3:



11.



$$y_s = \frac{\int_{-1}^{1} \frac{y}{2} \cdot y \, dx}{\text{Fläche}}$$

$$= \cancel{\frac{1}{2}} \, \frac{\int_{-1}^{1} \sqrt{1-x^2}^{\,2} \, dx}{\cancel{\frac{1}{2}} \pi}$$

$$= \frac{1}{\pi} \int_{-1}^{1} (1-x^2) \, dx$$

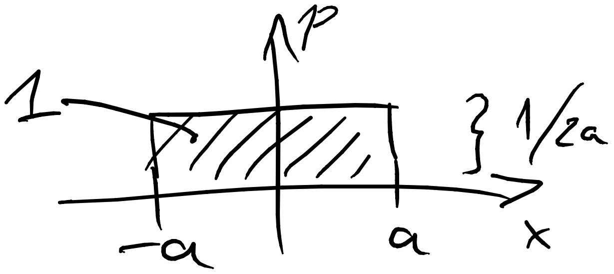
$$= \frac{1}{\pi} \left[x - \frac{x^3}{3} \right]_{-1}^{1} = \frac{1}{\pi} \left(1 - \frac{1}{3} - \left(-1 - \frac{(-1)^3}{3} \right) \right)$$

$$= \frac{4}{3\pi} \quad ((\approx 0{,}4))$$

12. Offensichtlich $E[X]=0$.

Also $\sigma^2 = E[X^2] - 0^2$

$$= \int_{-\infty}^{\infty} x^2 p(x) \, dx$$



$$=\frac{1}{2a}\int_{-a}^{a} x^2\,dx$$

$$=\frac{1}{2a}\left[\frac{x^3}{3}\right]_{-a}^{a}$$

$$=\frac{1}{2a}\left(\frac{a^3}{3}-\frac{(-a)^3}{3}\right)=\frac{\cancel{2}a^{\cancel{3}2}}{\cancel{2}\cancel{a}\cdot 3}=\frac{a^2}{3}$$

$$\Rightarrow \sigma = \frac{a}{\sqrt{3}}.$$

a muss $\sqrt{3}$ sein, damit das 1 wird.