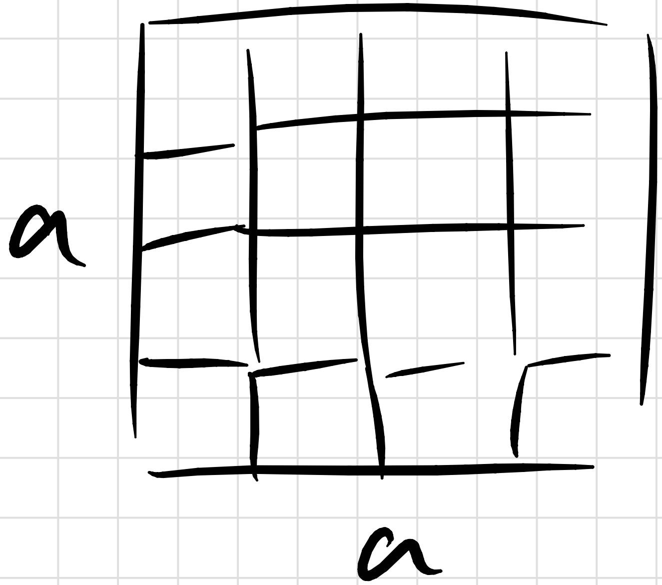
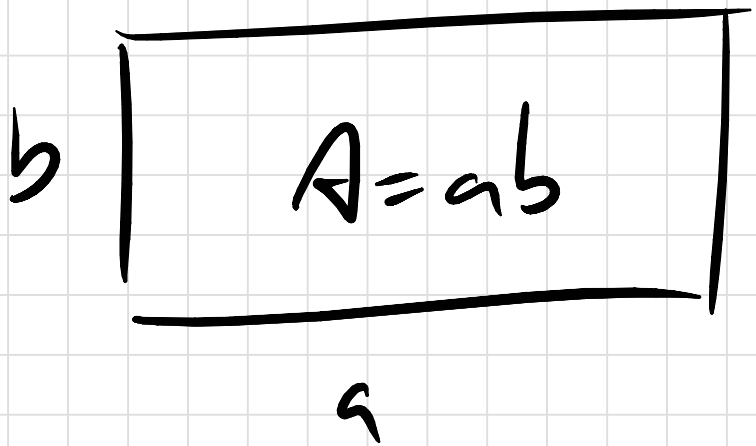
Quadrat

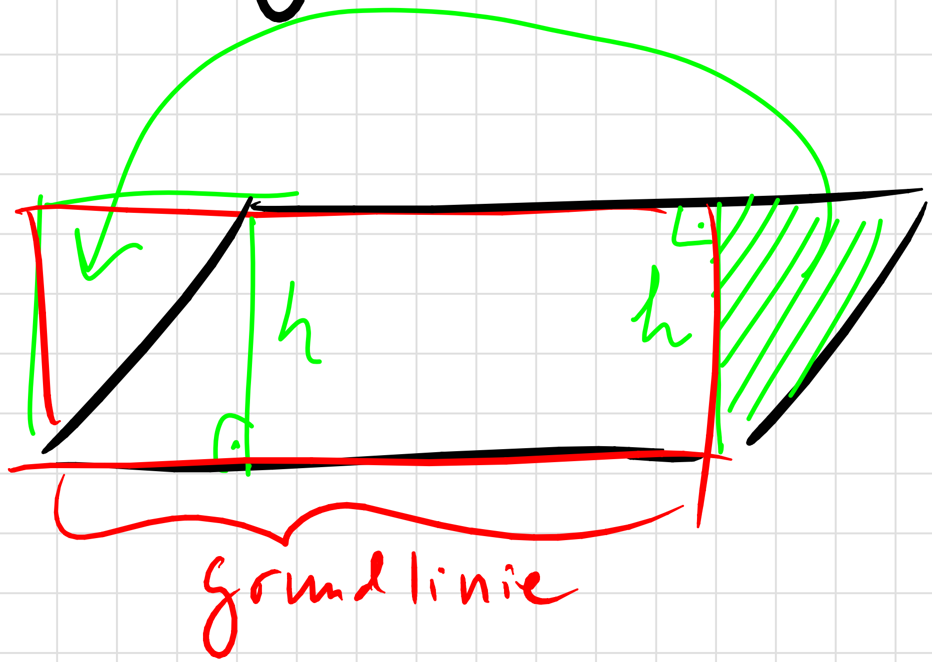


Fläche a^2

Rechteck

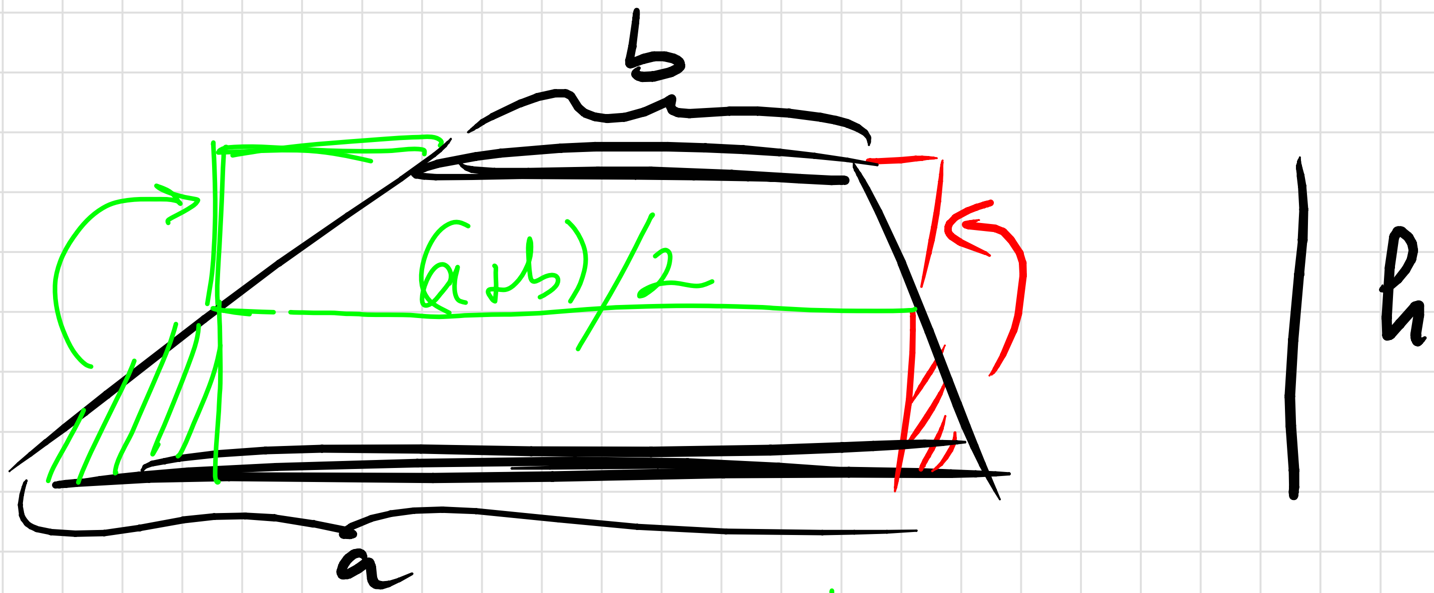


Parallelogramm



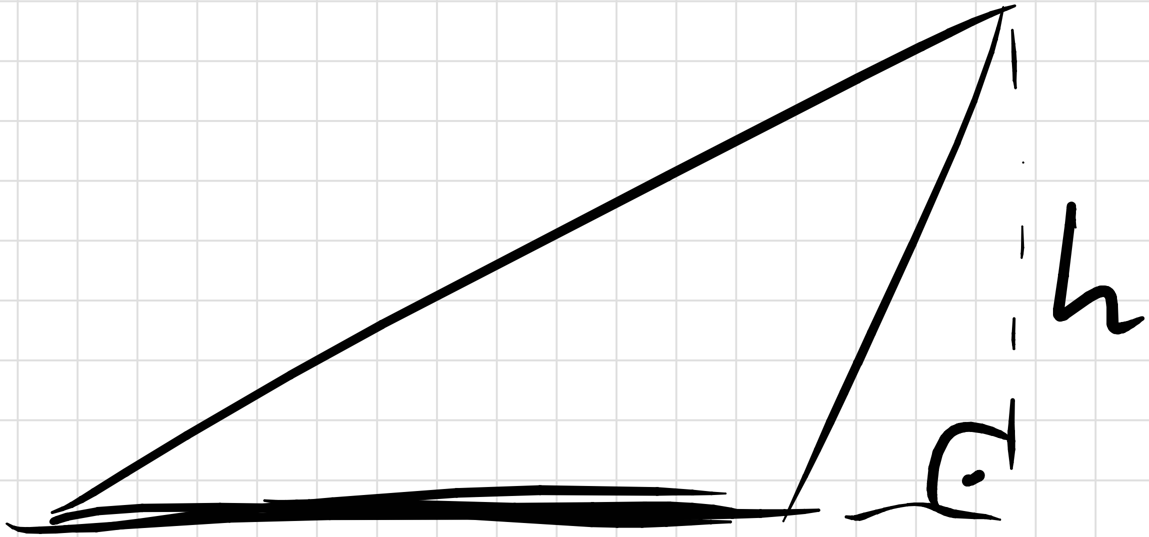
A = Grundlinie · Höhe

Trapez



$$A = h \cdot \frac{a+b}{2}$$

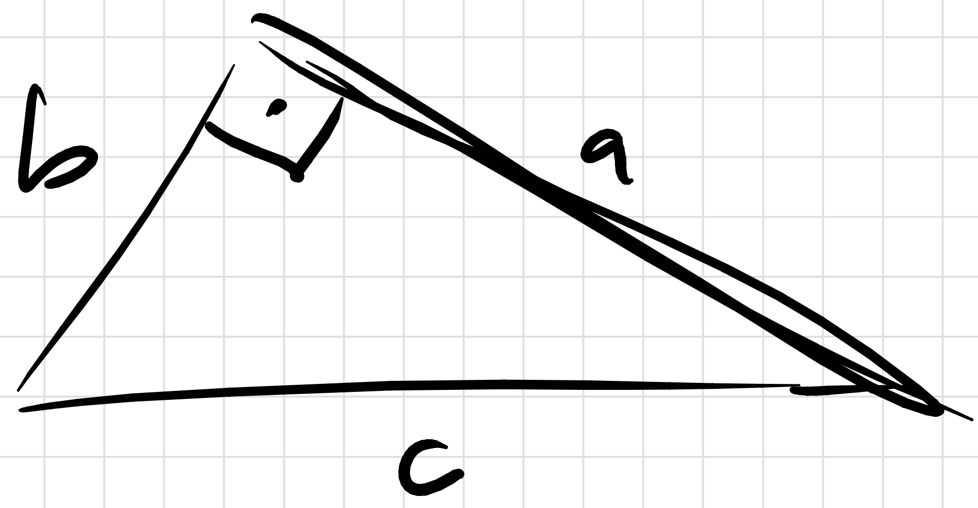
Dreieck



Grundlinie

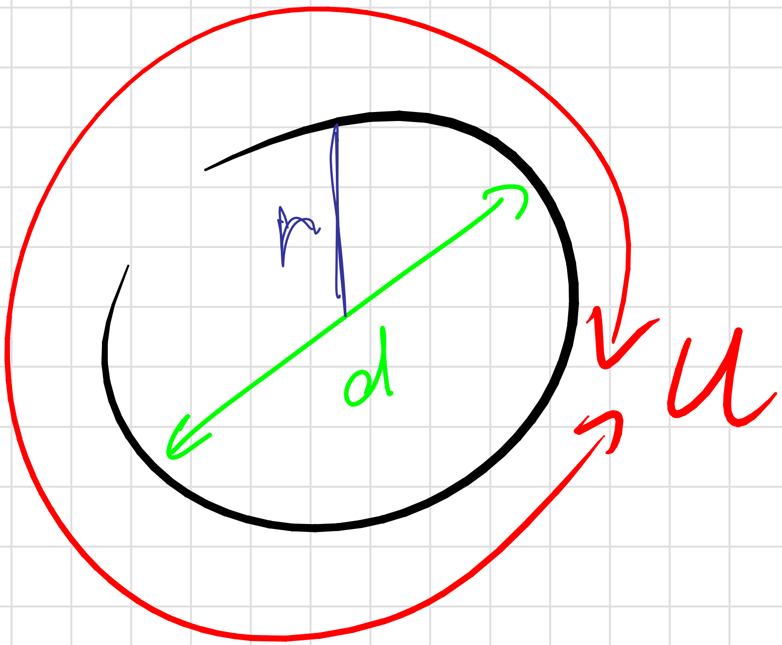
$$A = \frac{1}{2} \text{ Grundlinie} \cdot \text{Höhe}$$

rechtw. Dreieck



$$A = \frac{1}{2} a b$$

Kreisumfang

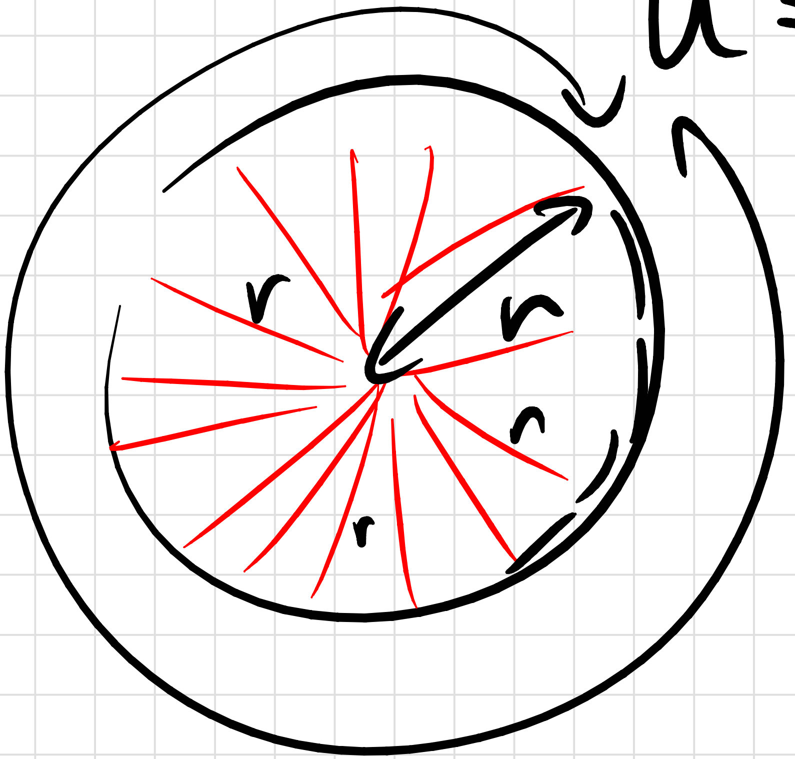


d	U
1 m	3,14 m
2 m	6,28 m

Proportionalität

π = Proportionalitäts-Konstante

$$U = \pi d$$
$$= 2\pi r$$

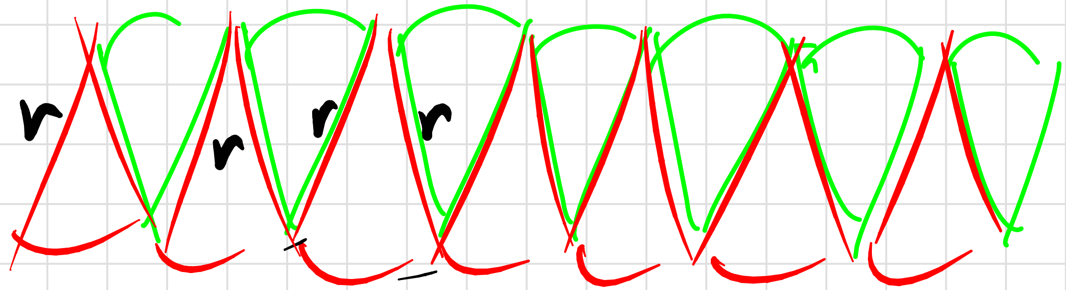


$$U = 2\pi r$$

$$A = \frac{1}{2} r U$$

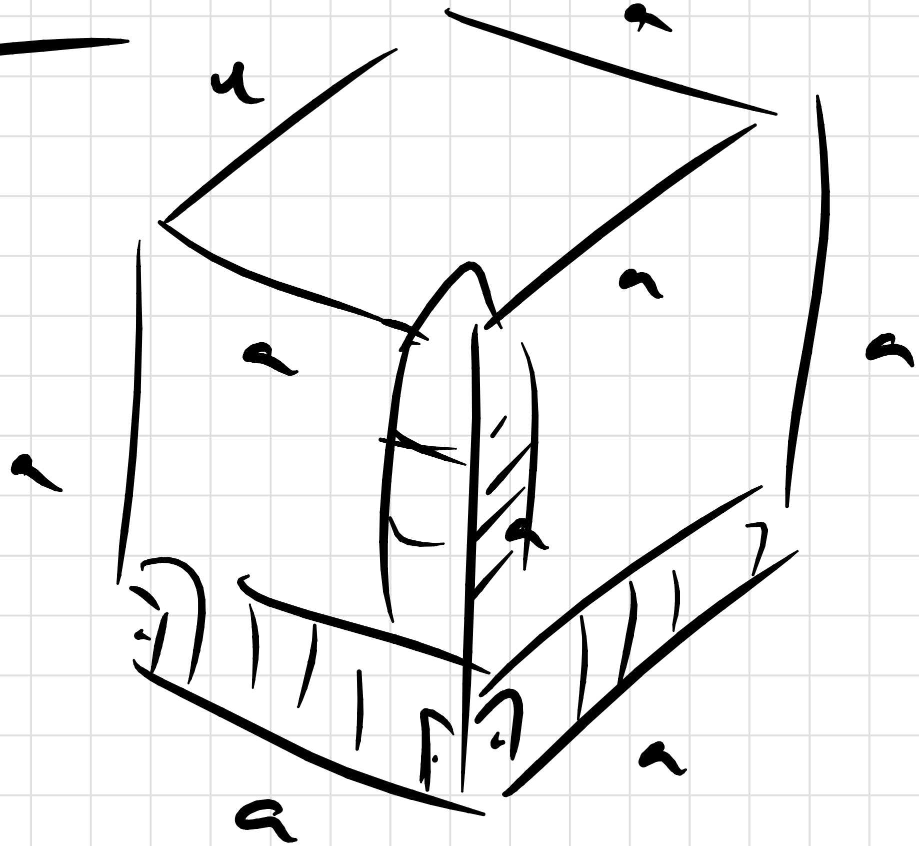
$$= \frac{1}{\cancel{2}} r \, \cancel{2}\pi r$$

$$= \pi r^2$$



Volumen

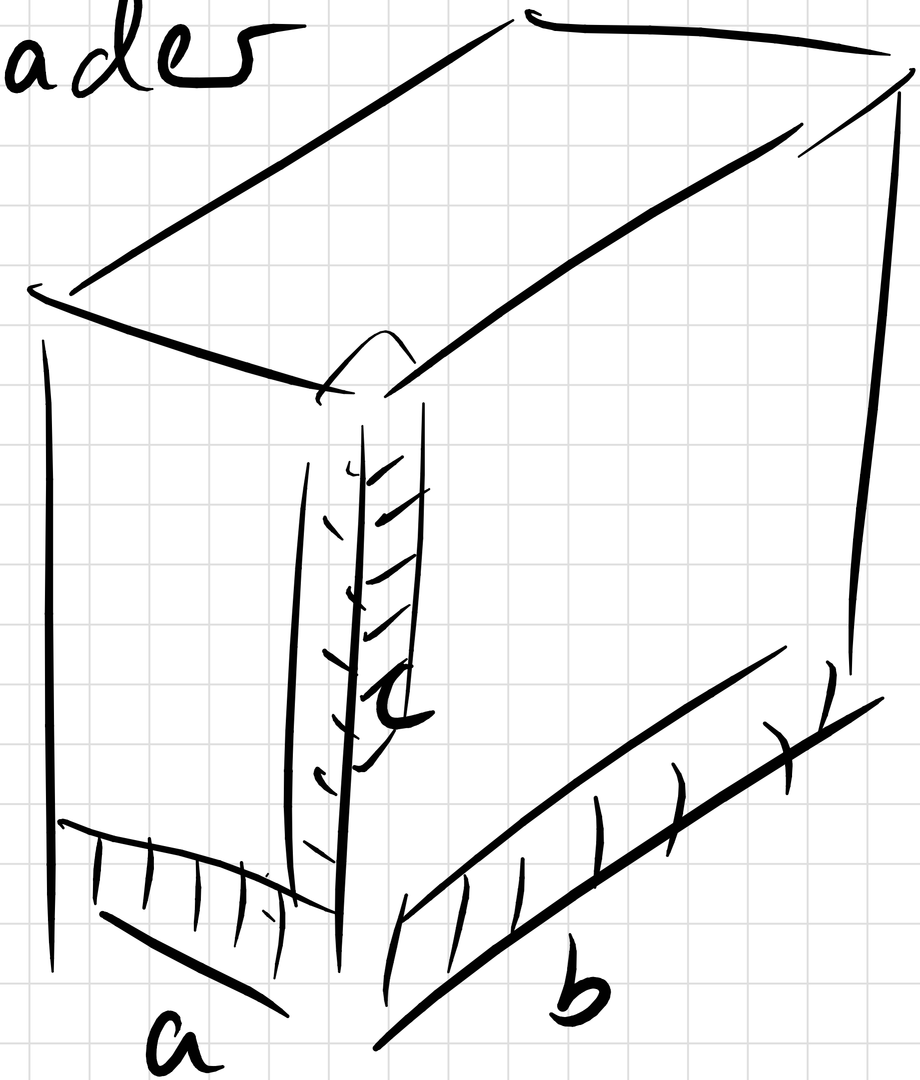
Würfel



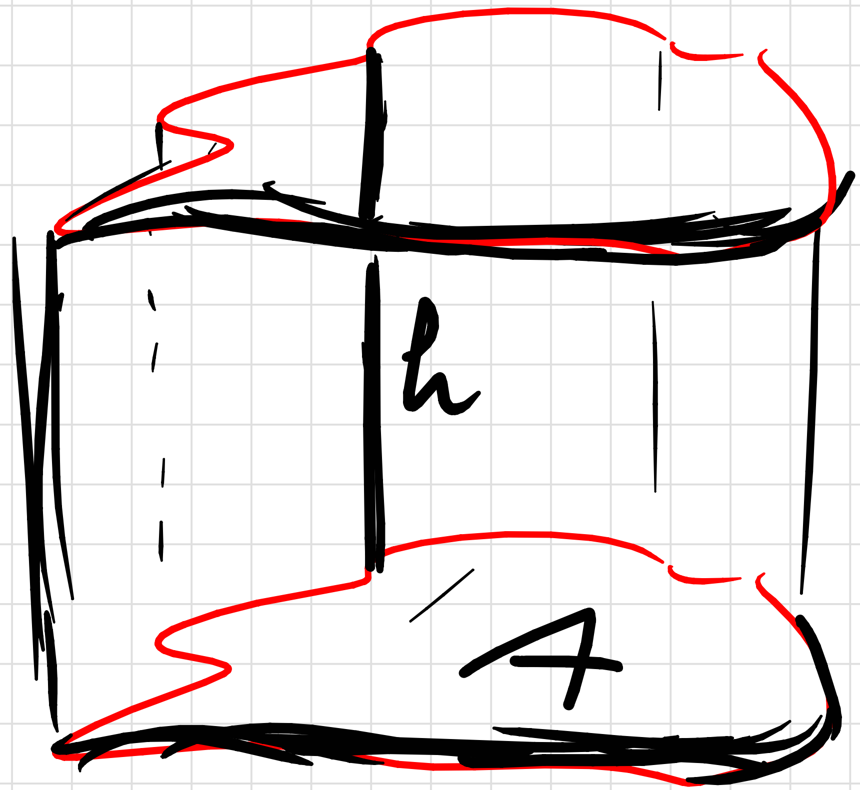
$$V = a^3$$

Quader

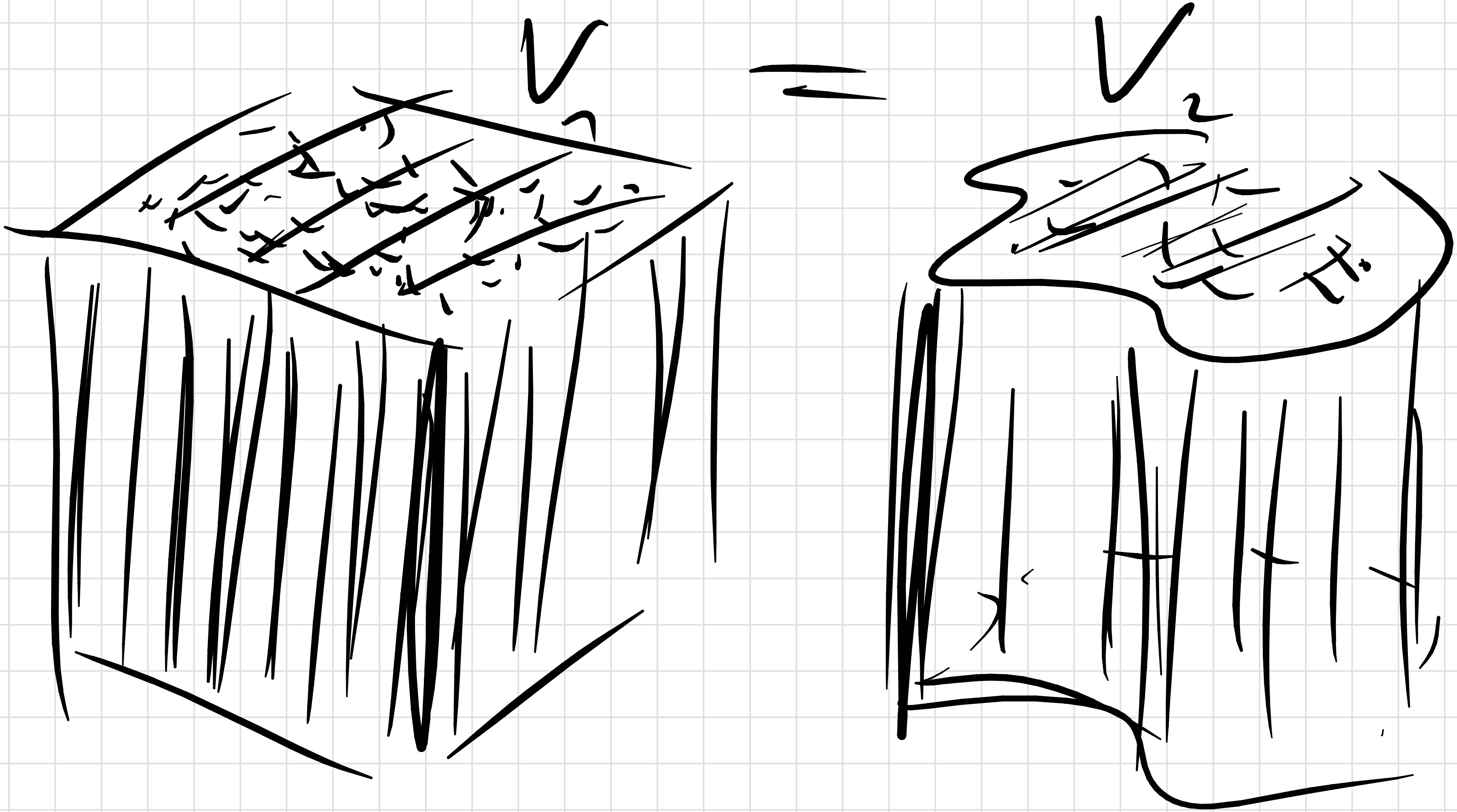
$V = a \cdot b \cdot c$



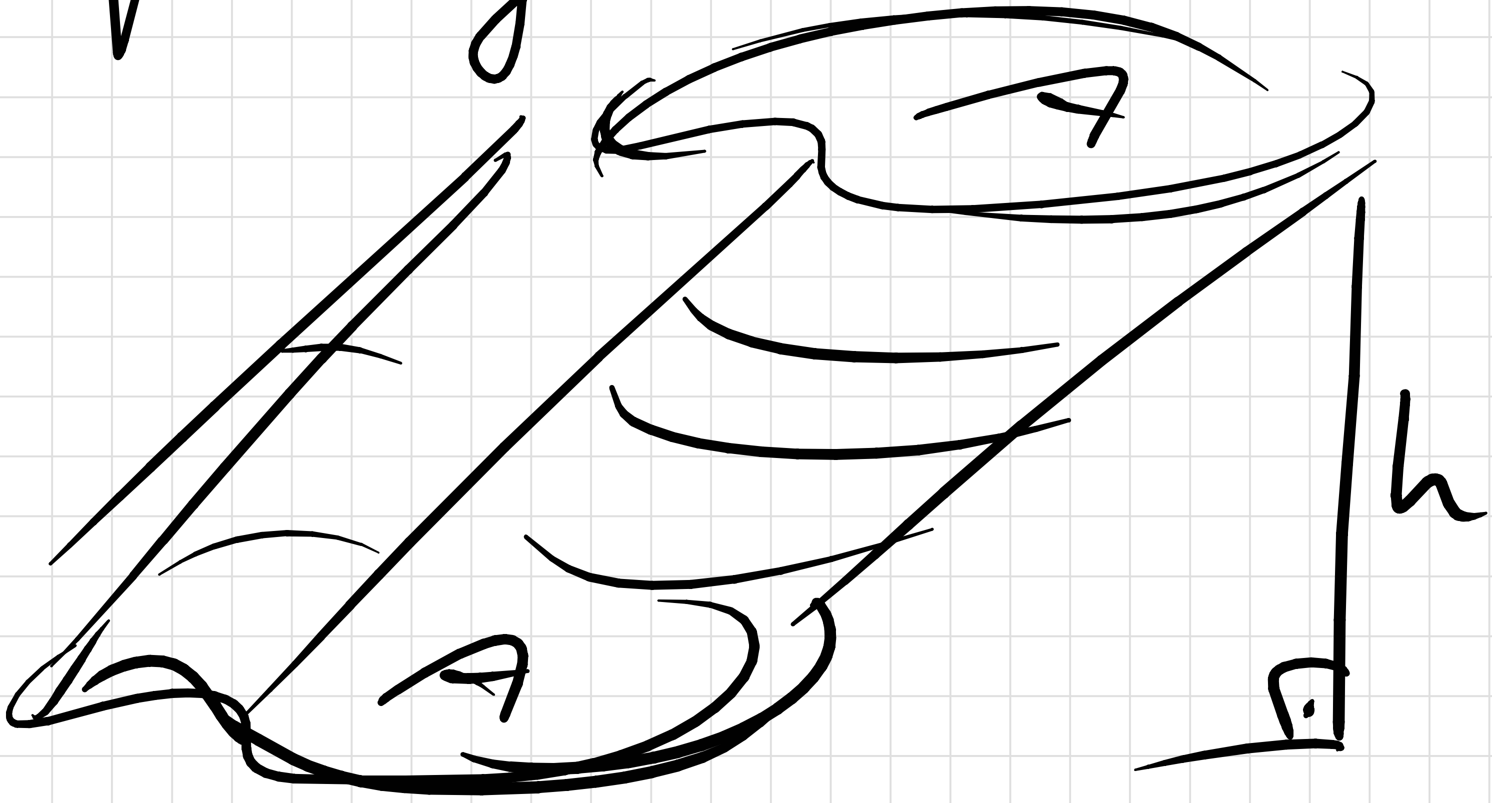
Gerader Zylinder



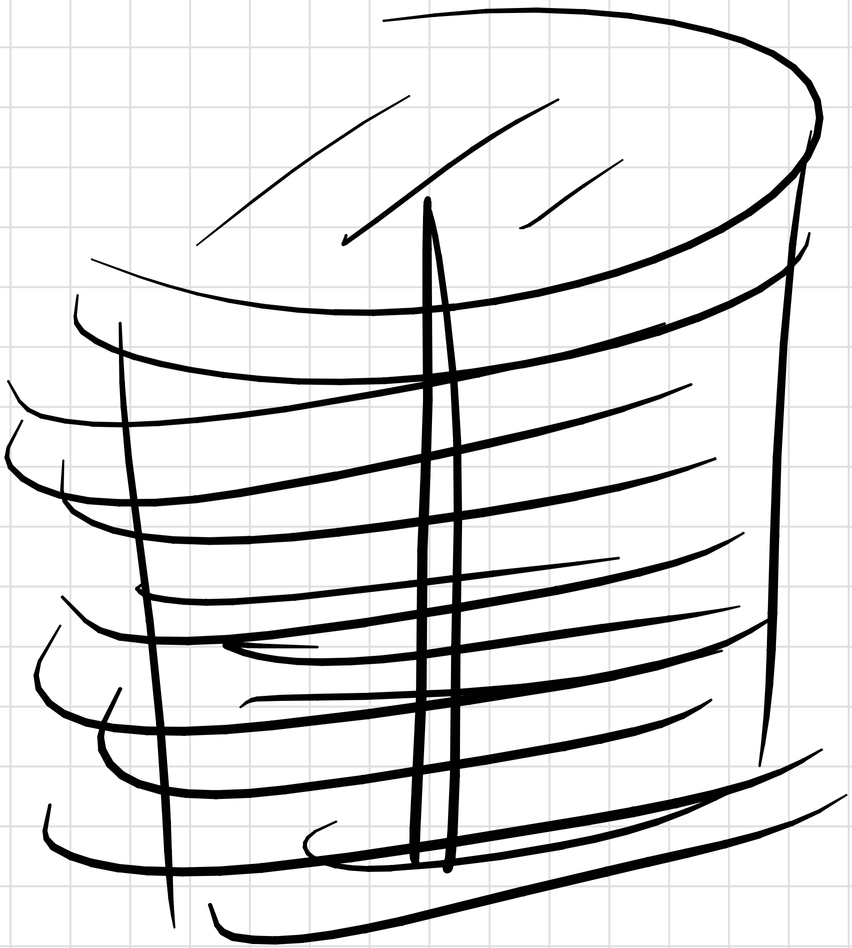
$$V = A \cdot h$$

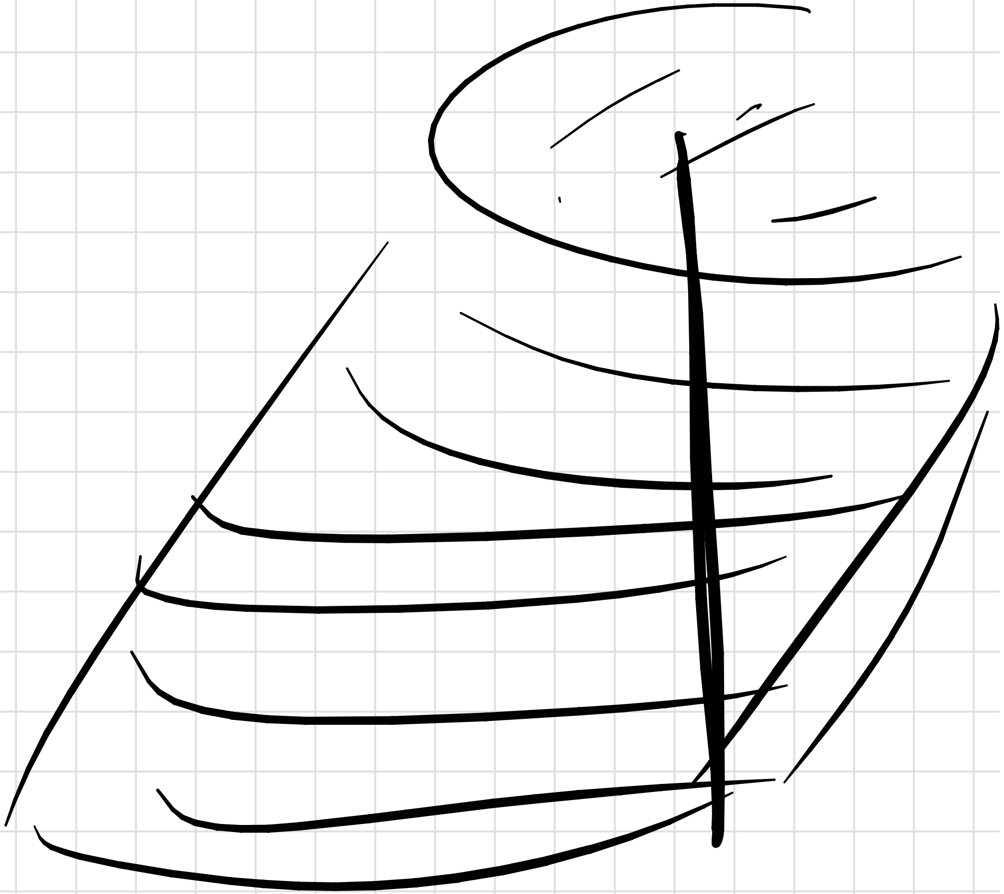

$V_1 = V_2$

Schiefer Zylinder

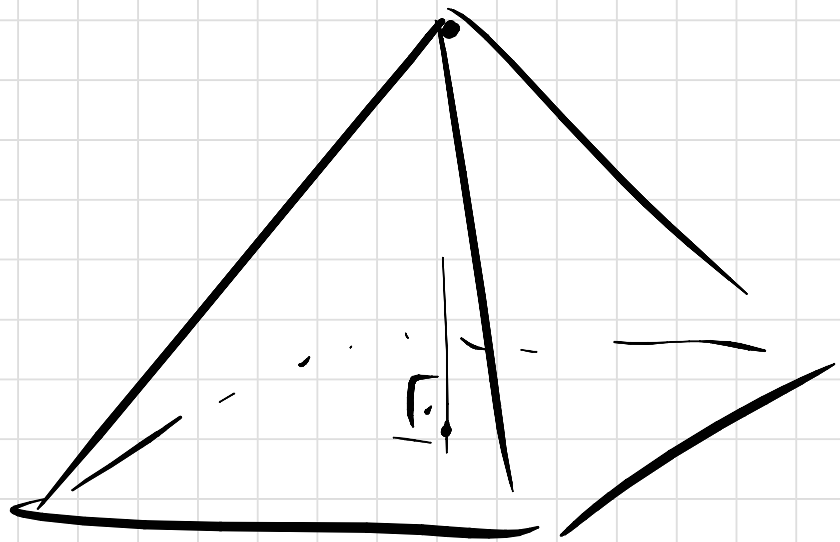


$$V = A \cdot h$$

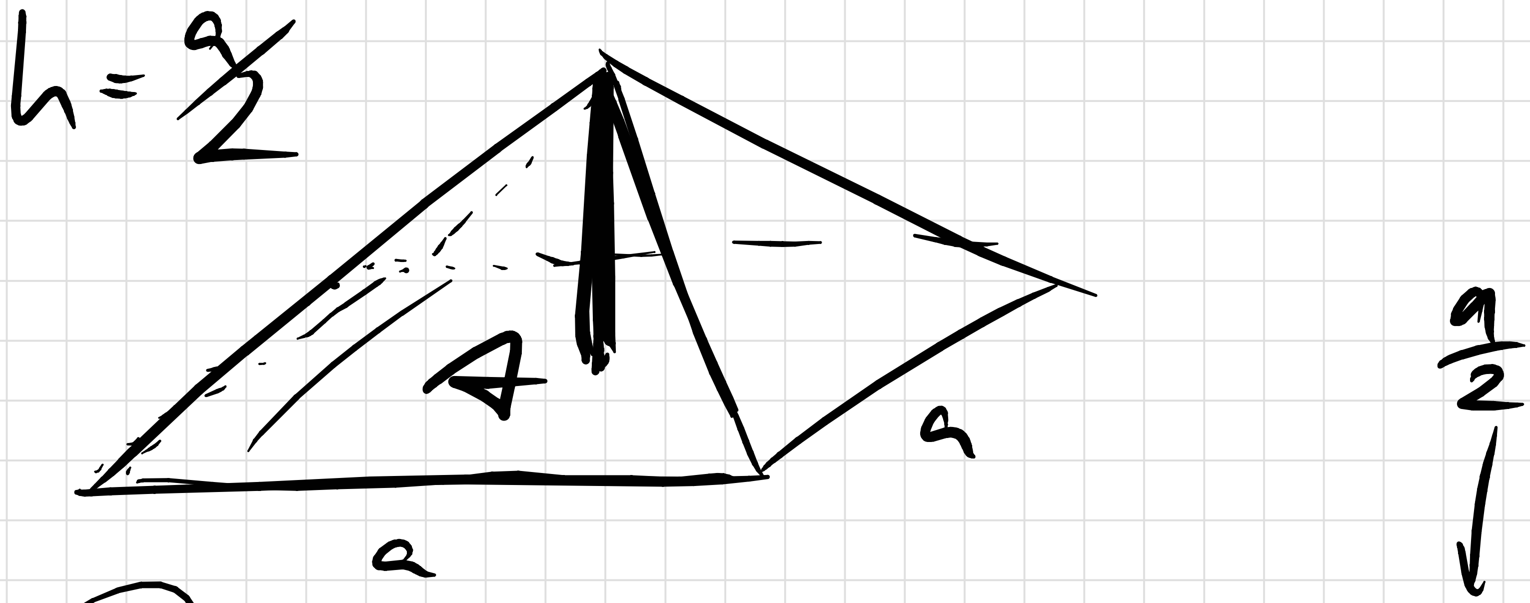




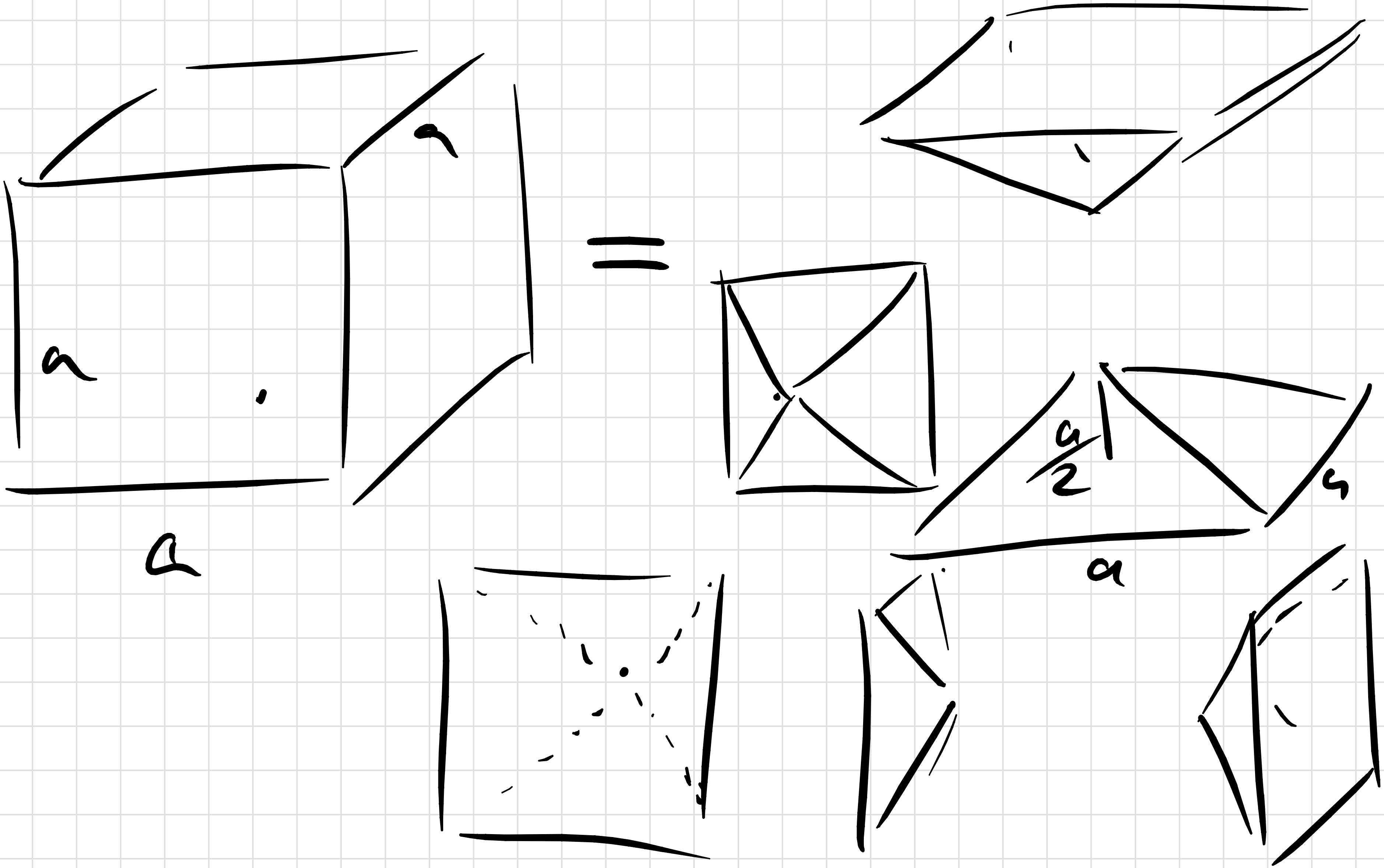
quadratische Pyramide

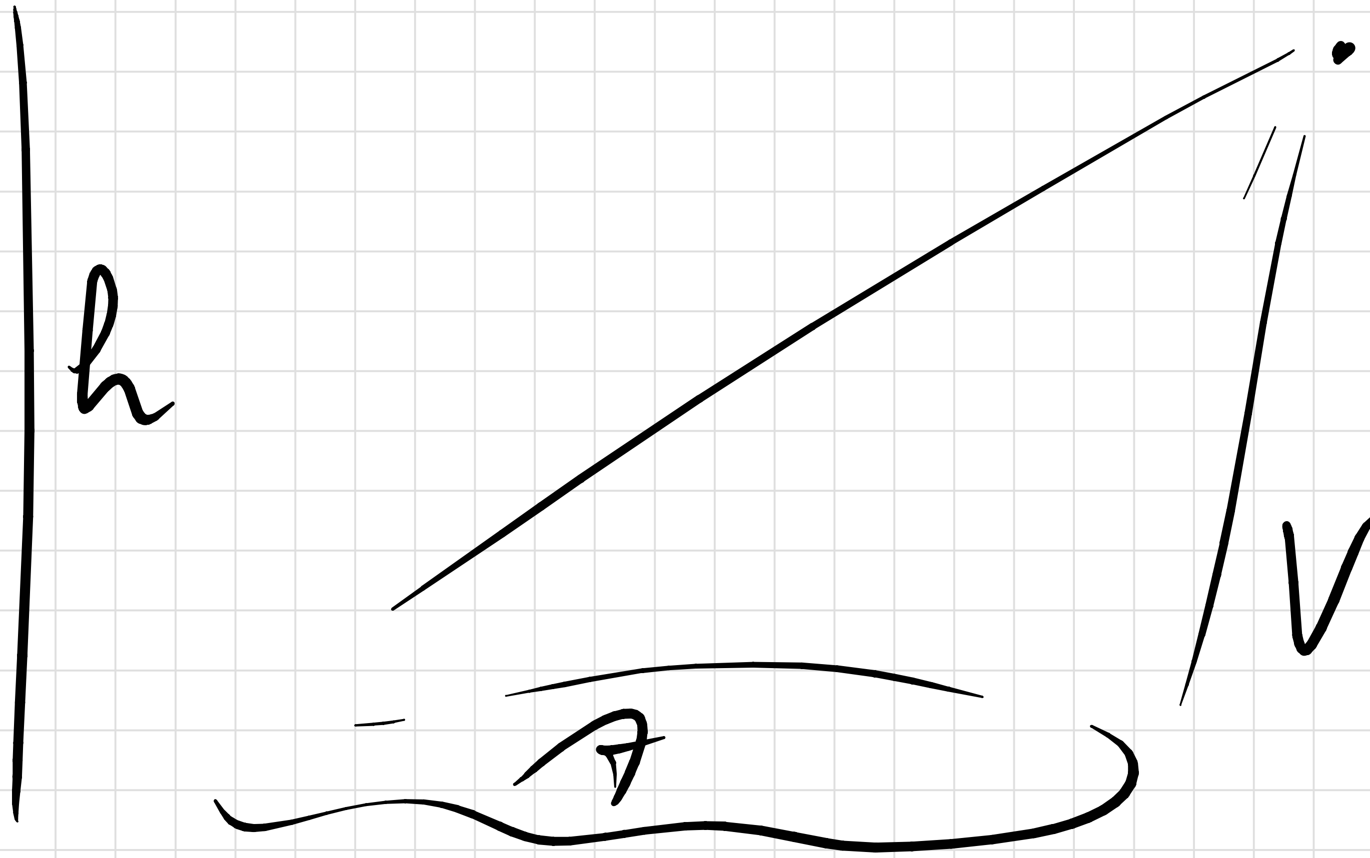


noch spezieller :



$$V = a^3/6 = \frac{A}{3} \cdot h$$


a
a
a
=
a/2
a
a



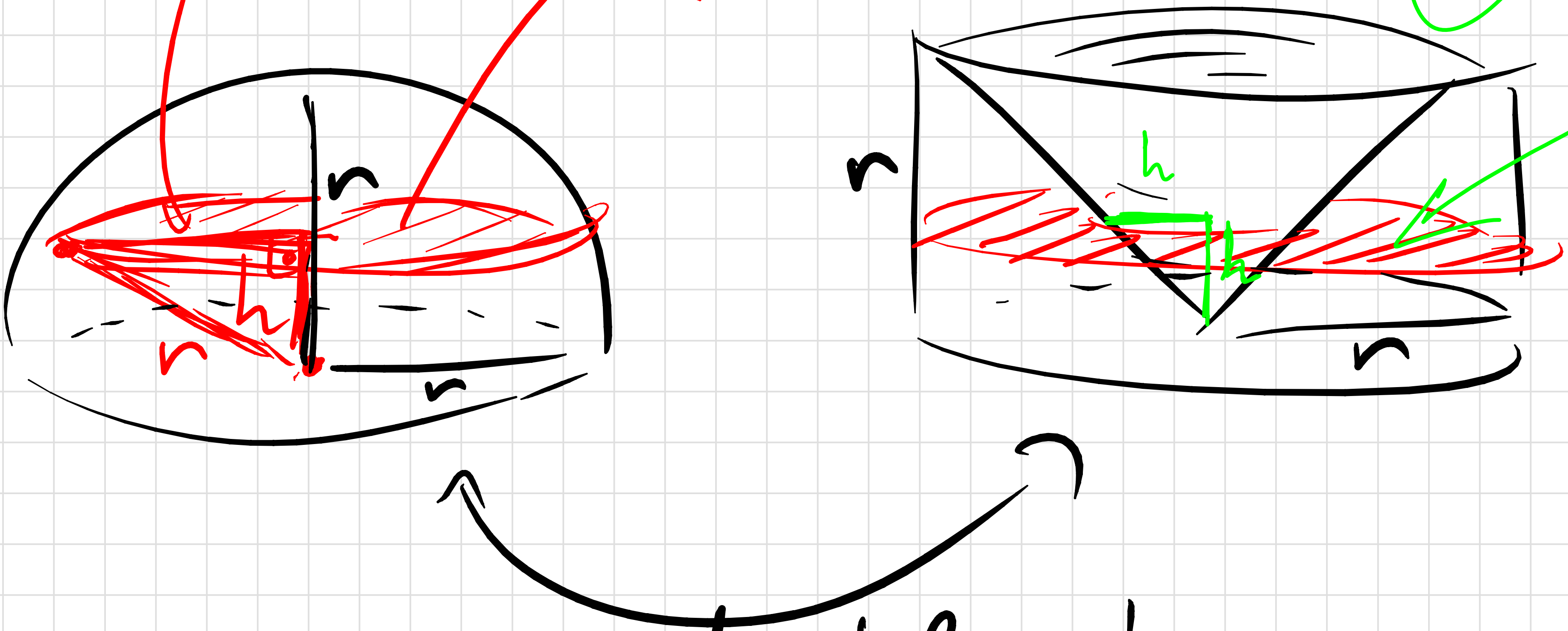
Kegel

$$V = \frac{1}{3} A \cdot h$$

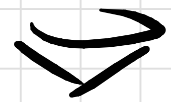
Kugelvolumen

Radius $= \sqrt{r^2 - h^2}$

$\pi\left(\sqrt{r^2-h^2}\right)^2 = \pi\left(r^2-h^2\right) = \pi r^2 - \pi h^2$



gleiches Volumen!

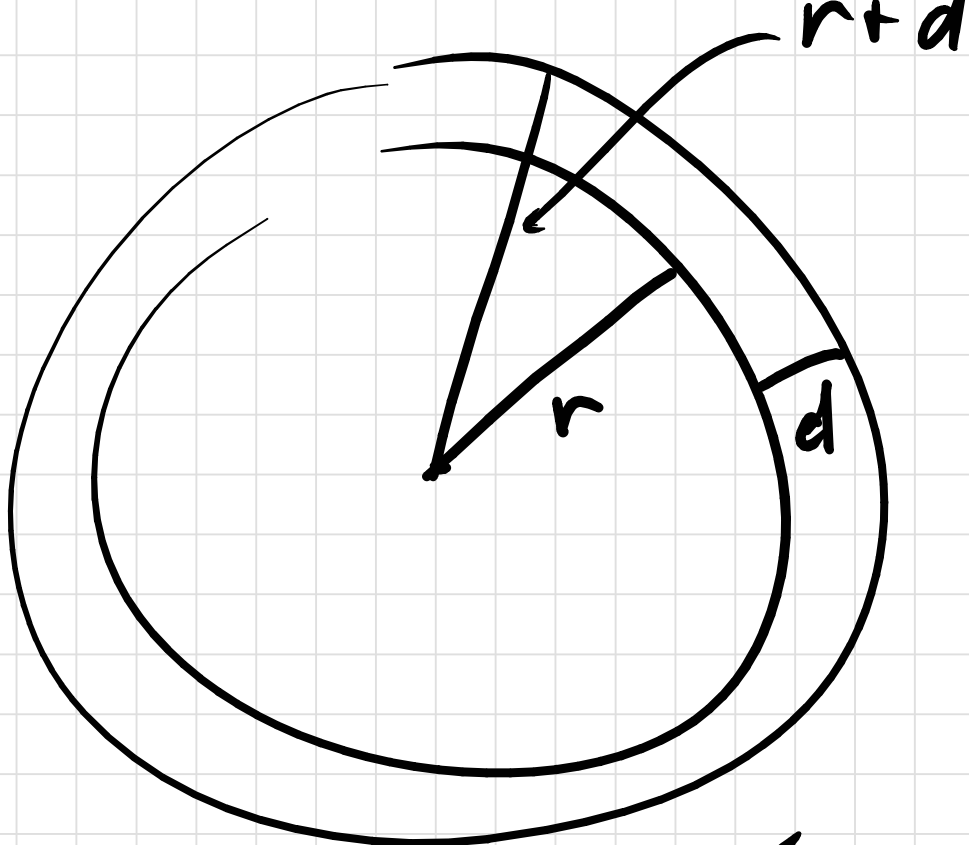
Kugelvolumen $= 2 \cdot ($  $-$  $)$

$$= 2 \cdot \left(\pi r^2 \cdot r - \frac{1}{3} \pi r^2 \cdot r \right)$$

$$= 2 \cdot \frac{2}{3} \pi \cdot r^3 = \frac{4}{3} \pi r^3$$

Lackiere eine Kugel
mit Radius r mit einer
Lackschicht der Dicke d.

Welches Volumen an Lack?



$V_{Lack} =$

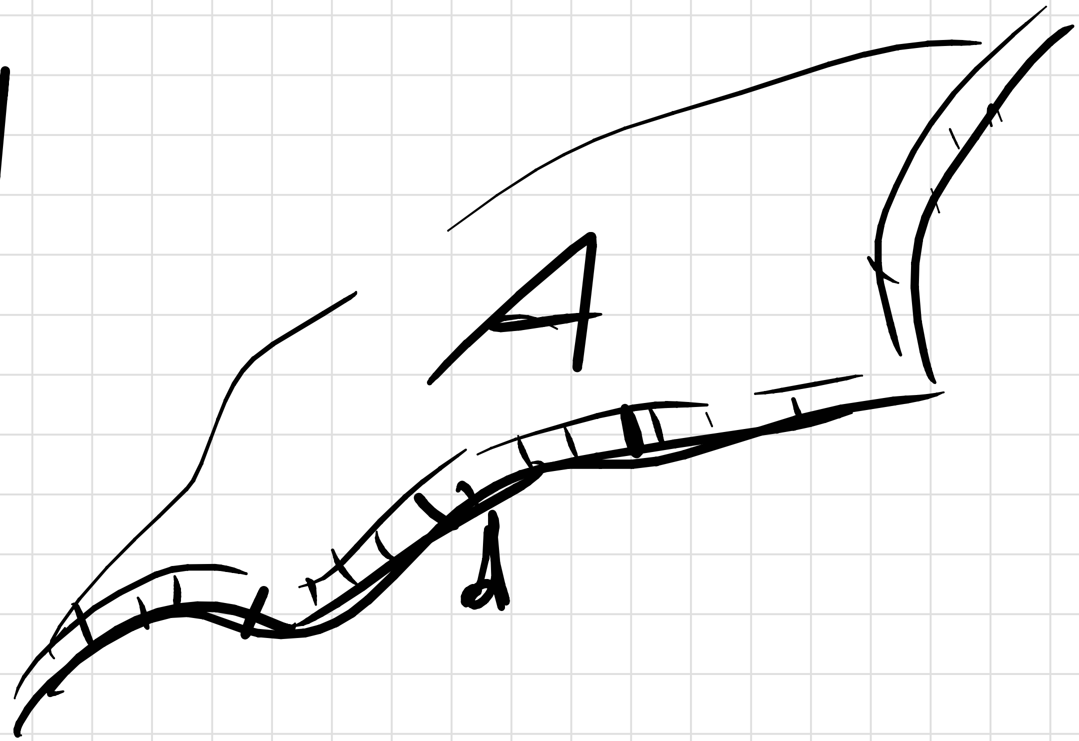
$\frac{4}{3}\pi (r+d)^3$

$- \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \left(\cancel{r^3} + 3r^2 d + \underbrace{3rd^2 + d^3}_{\text{vernachlässigbar}} - \cancel{r^3}\right)$

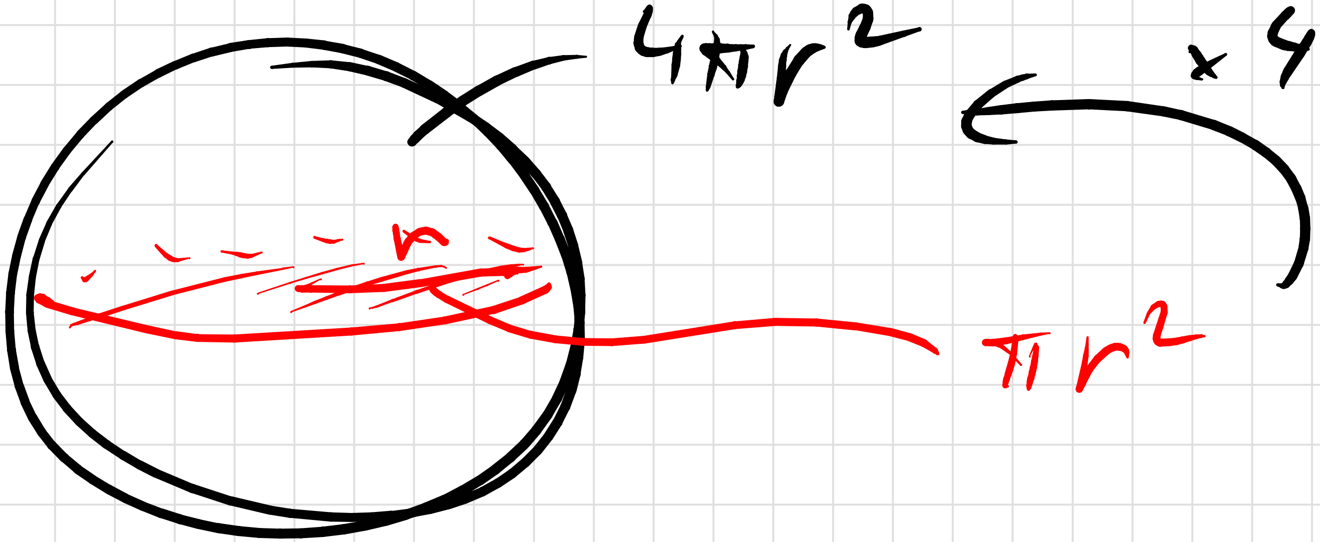
$$\approx \frac{4}{\cancel{3}}\pi \cdot \cancel{3} r^2 d = 4\pi r^2 d$$

$$= A \cdot d$$

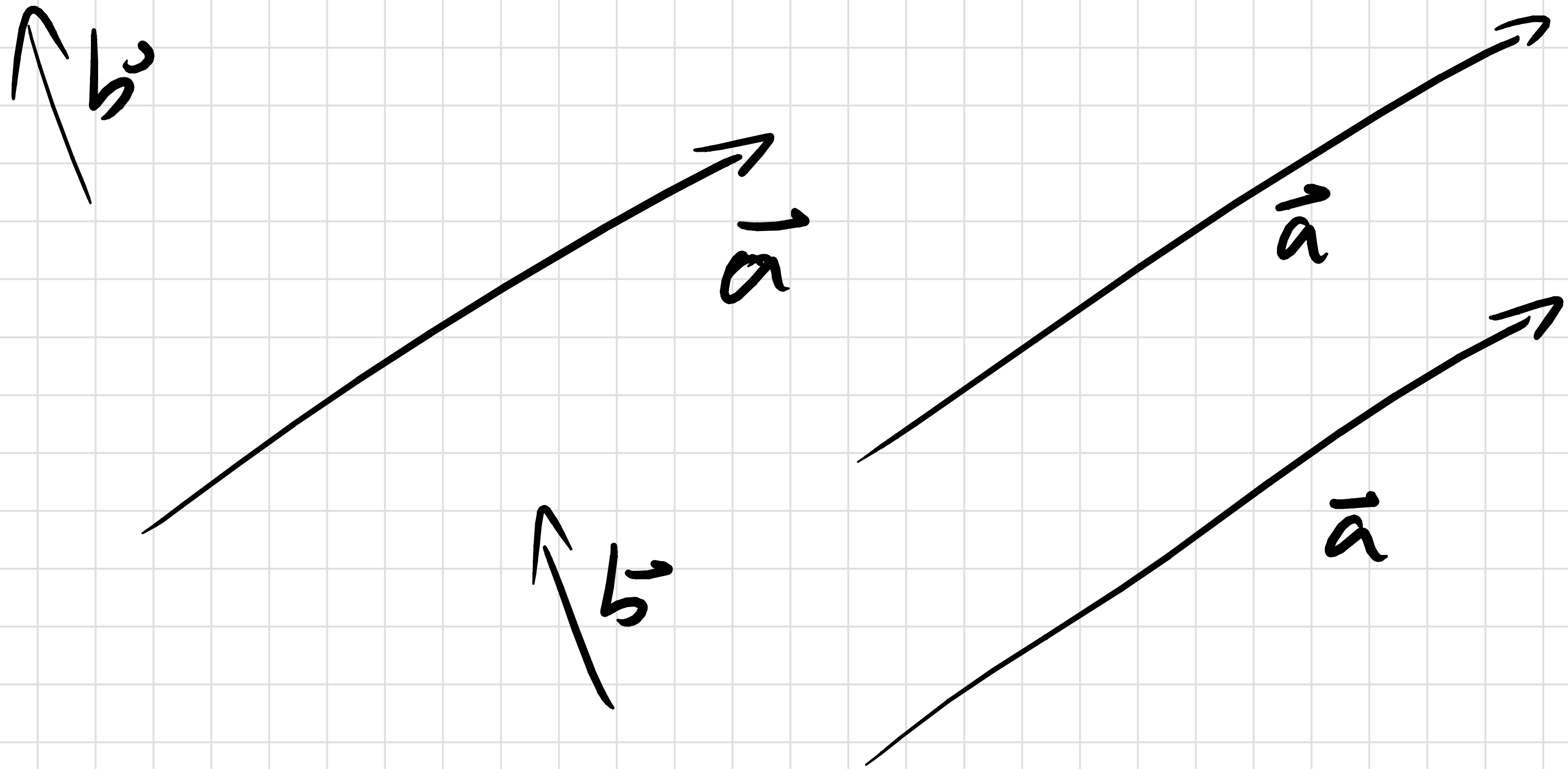


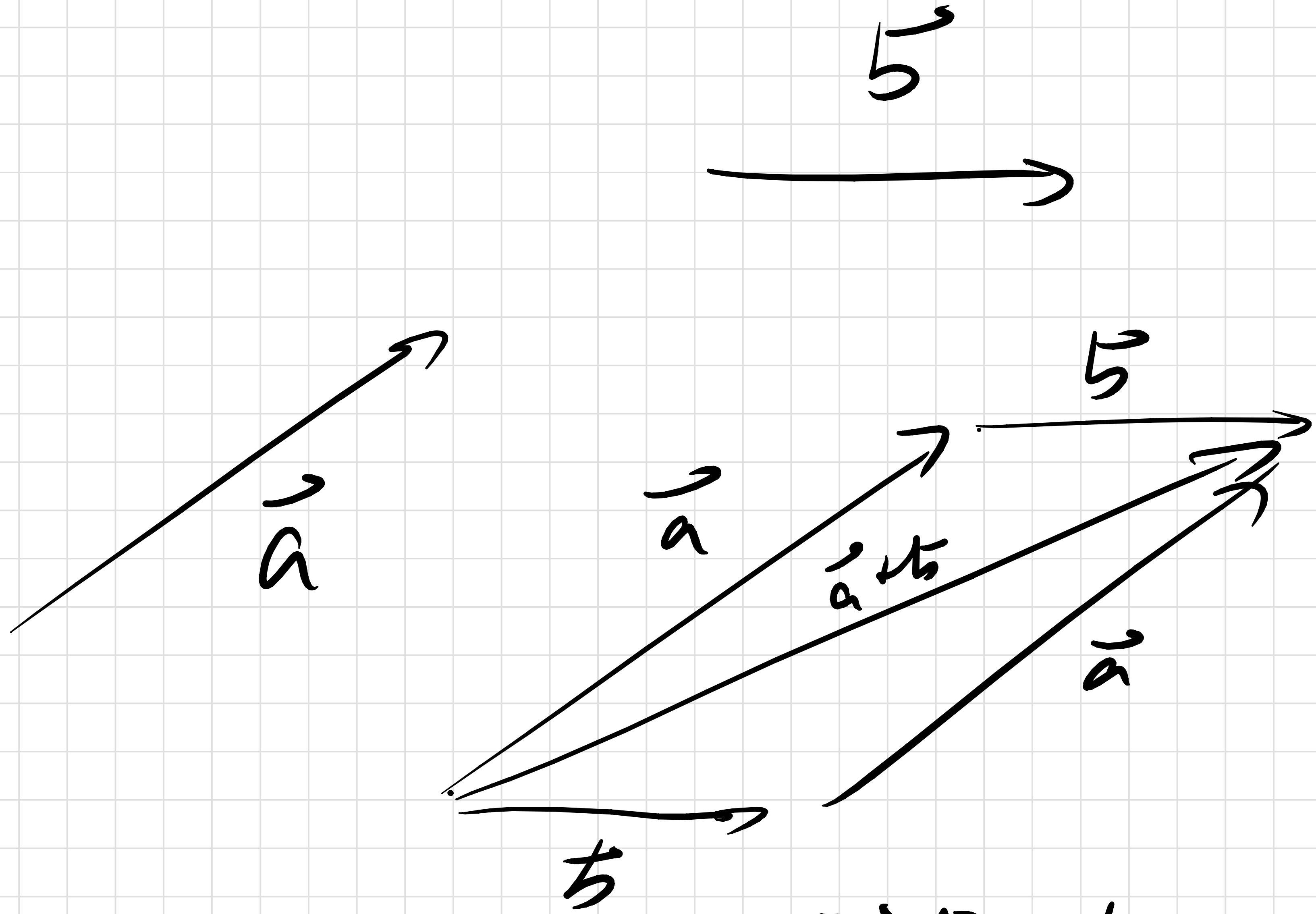
Also

$$A = 4\pi r^2$$

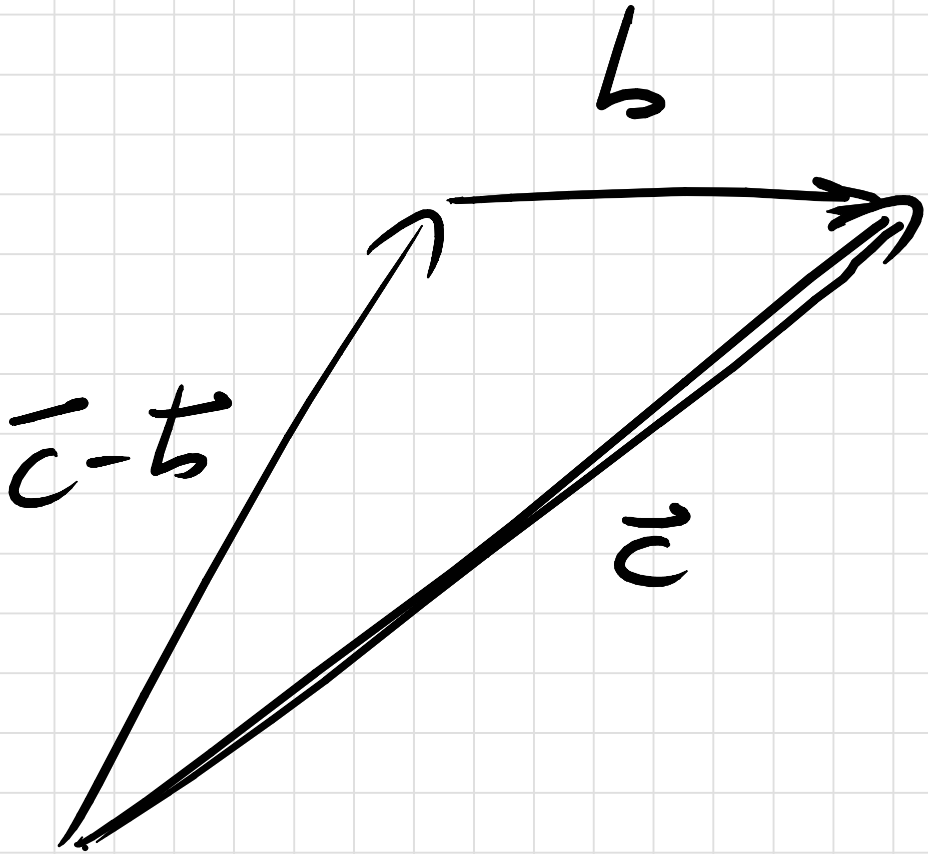

$4\pi r^2$
$\times 4$
r
πr^2

Vektoren

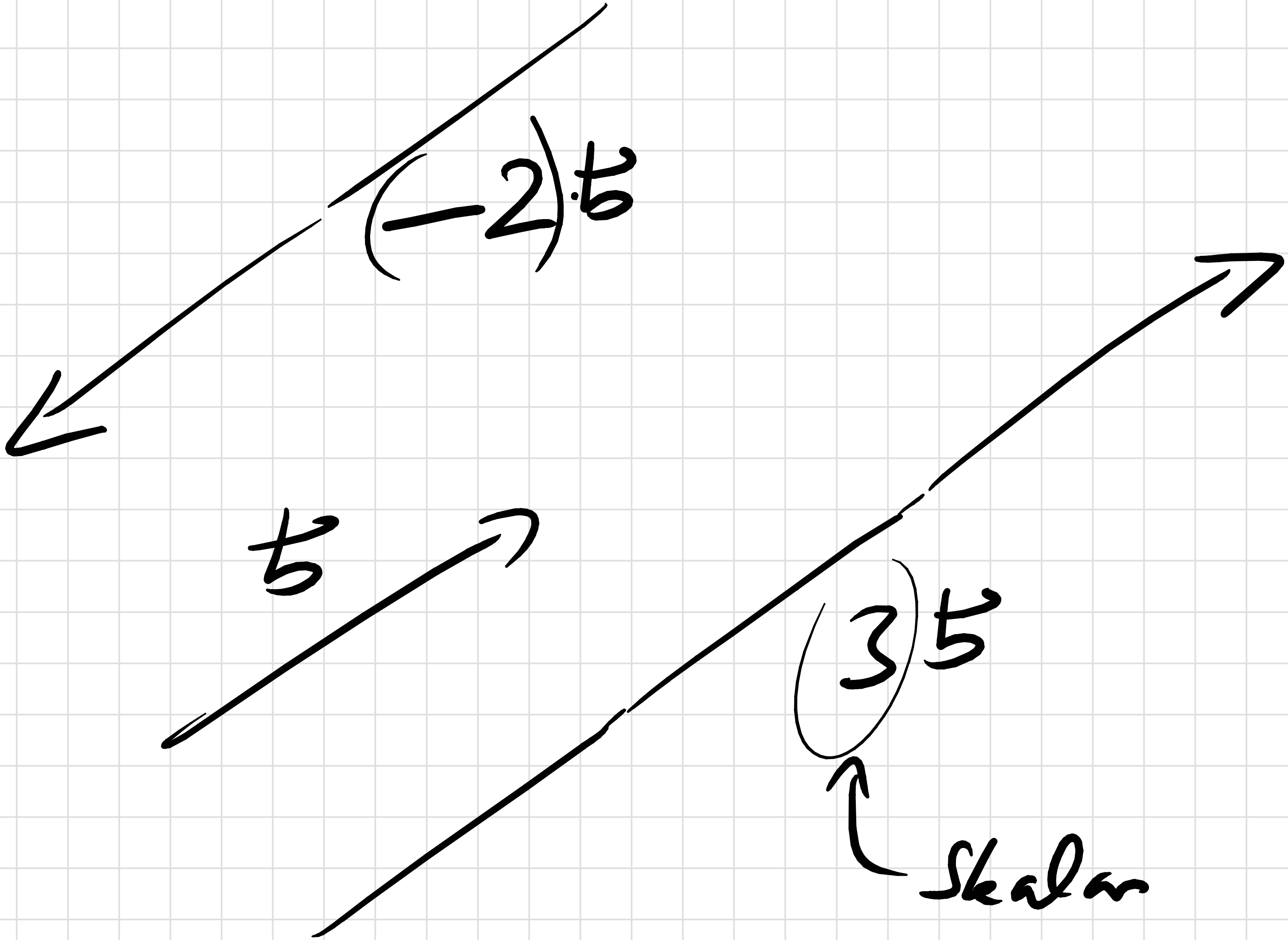




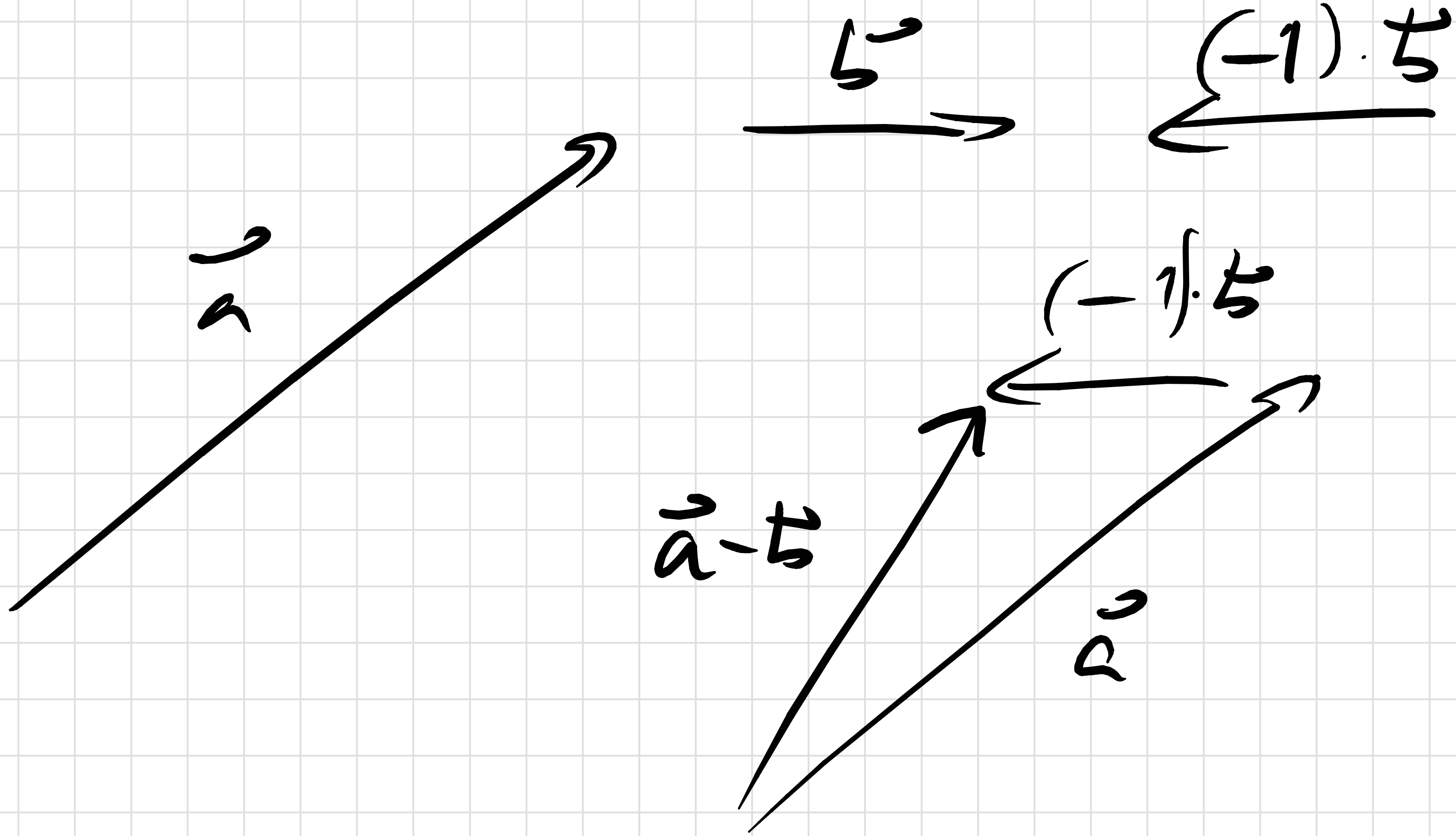
$$\vec{a}+\vec{b} = \vec{b}+\vec{a}$$



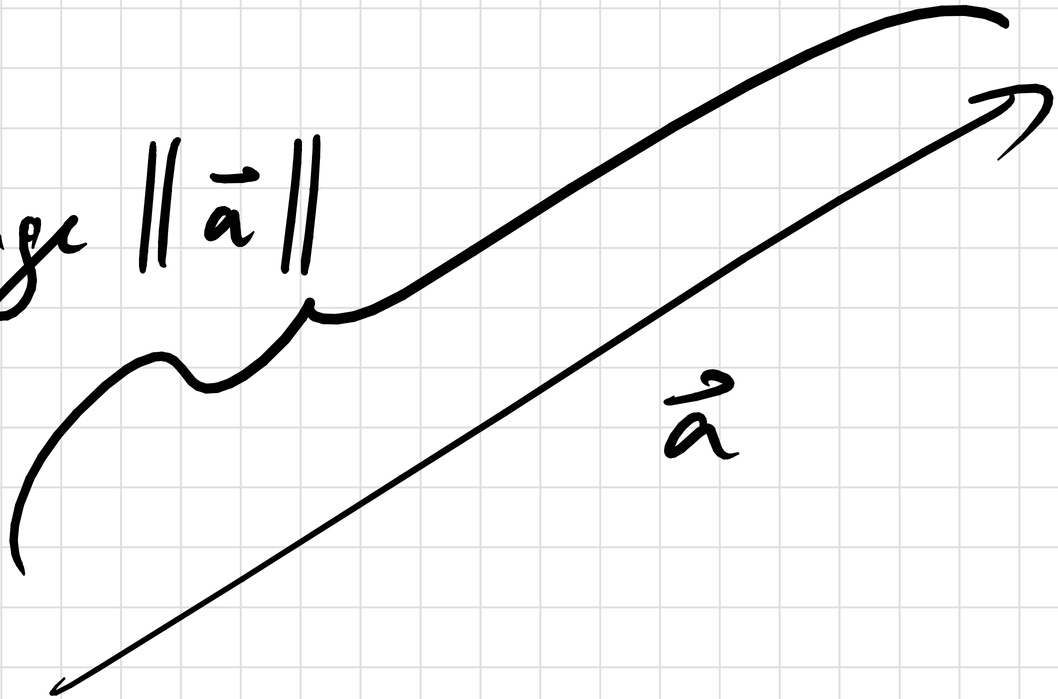
b
$\vec{c} - \vec{b}$
$\vec{c}$


$(-2)\cdot b$
b
$(3)b$
Skalar

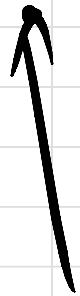
$$\vec{a} - \vec{b} = \vec{a} + (-1) \cdot \vec{b}$$



Länge $\|\vec{a}\|$

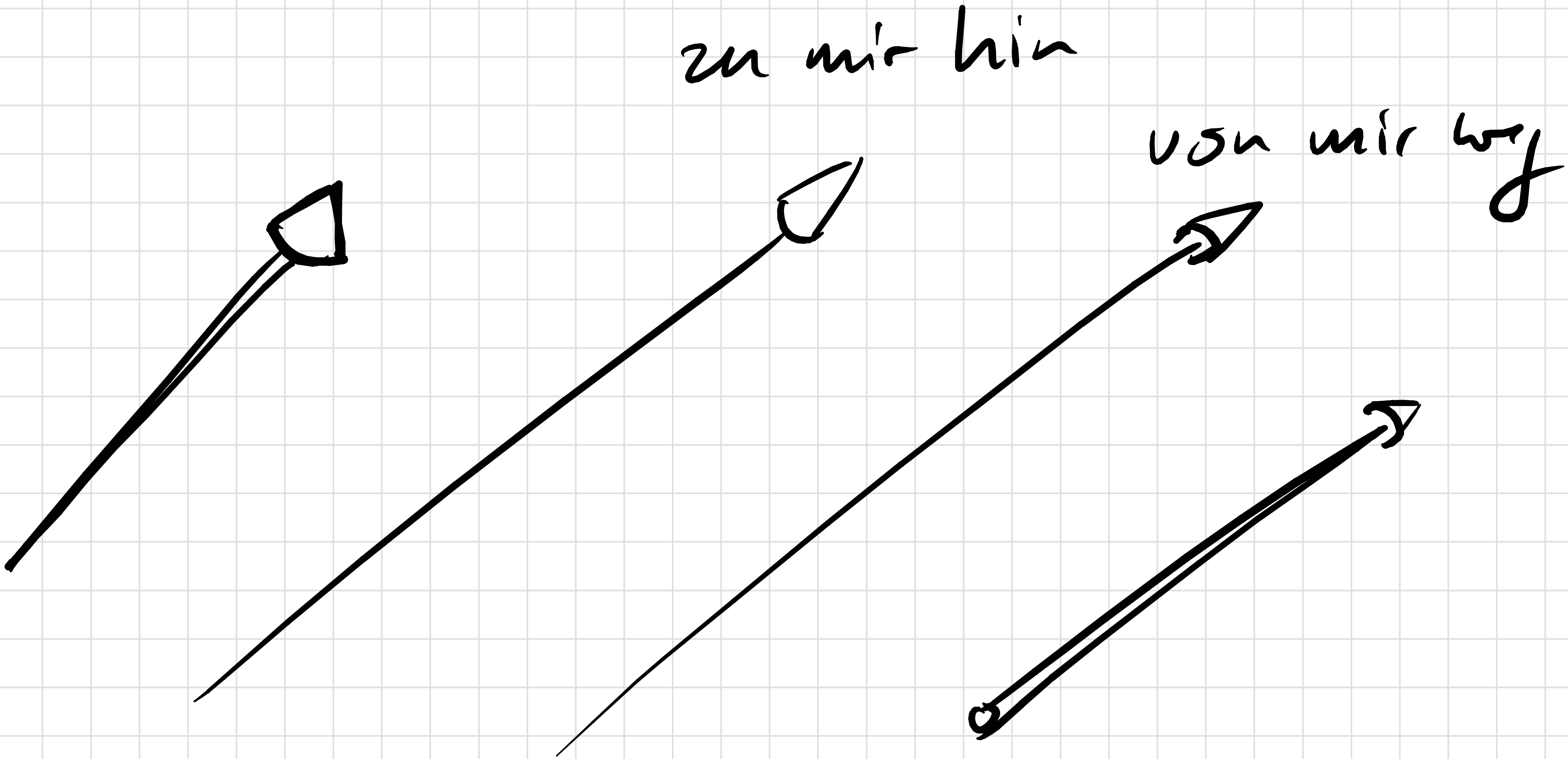


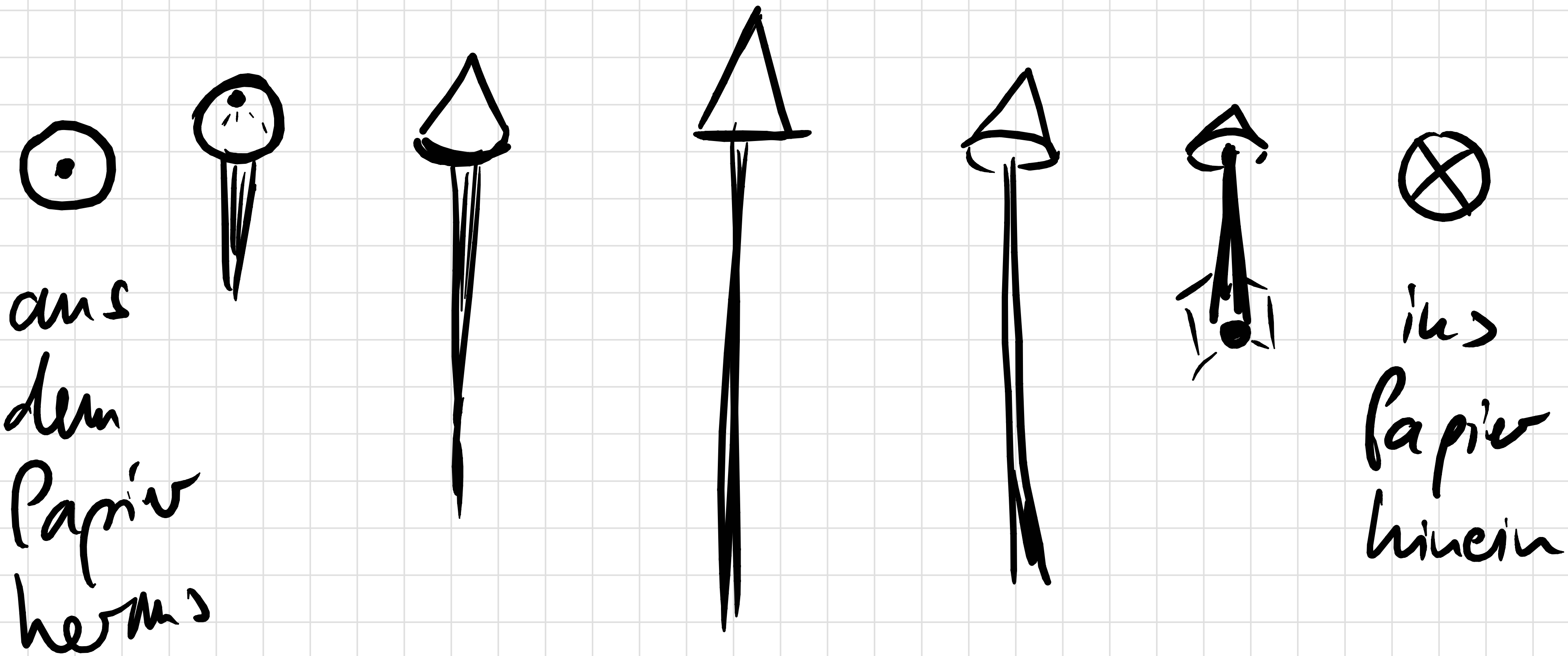
Nullvektor $\vec{0}$

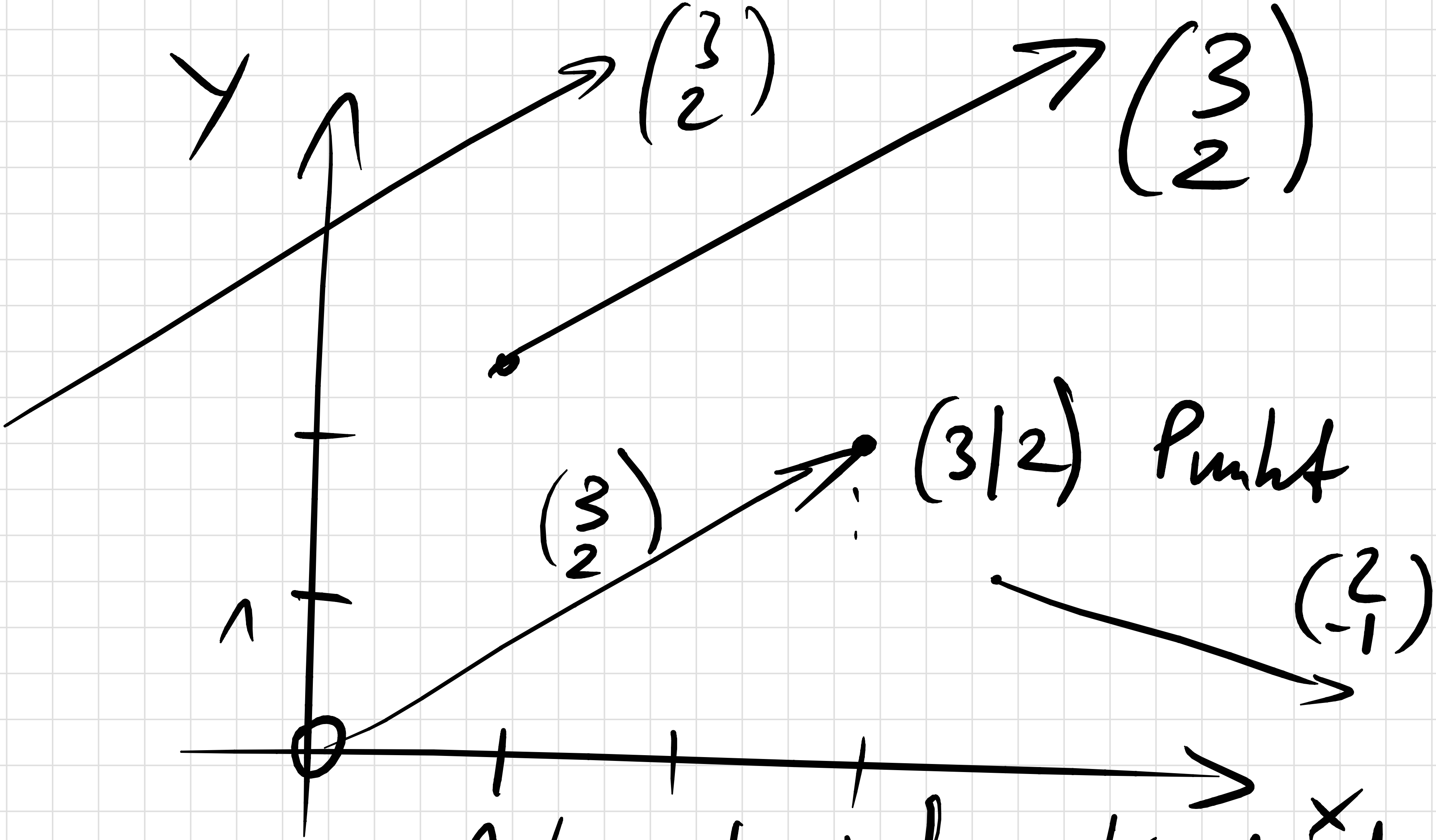


$\vec{b} = \vec{b} + \vec{0}$

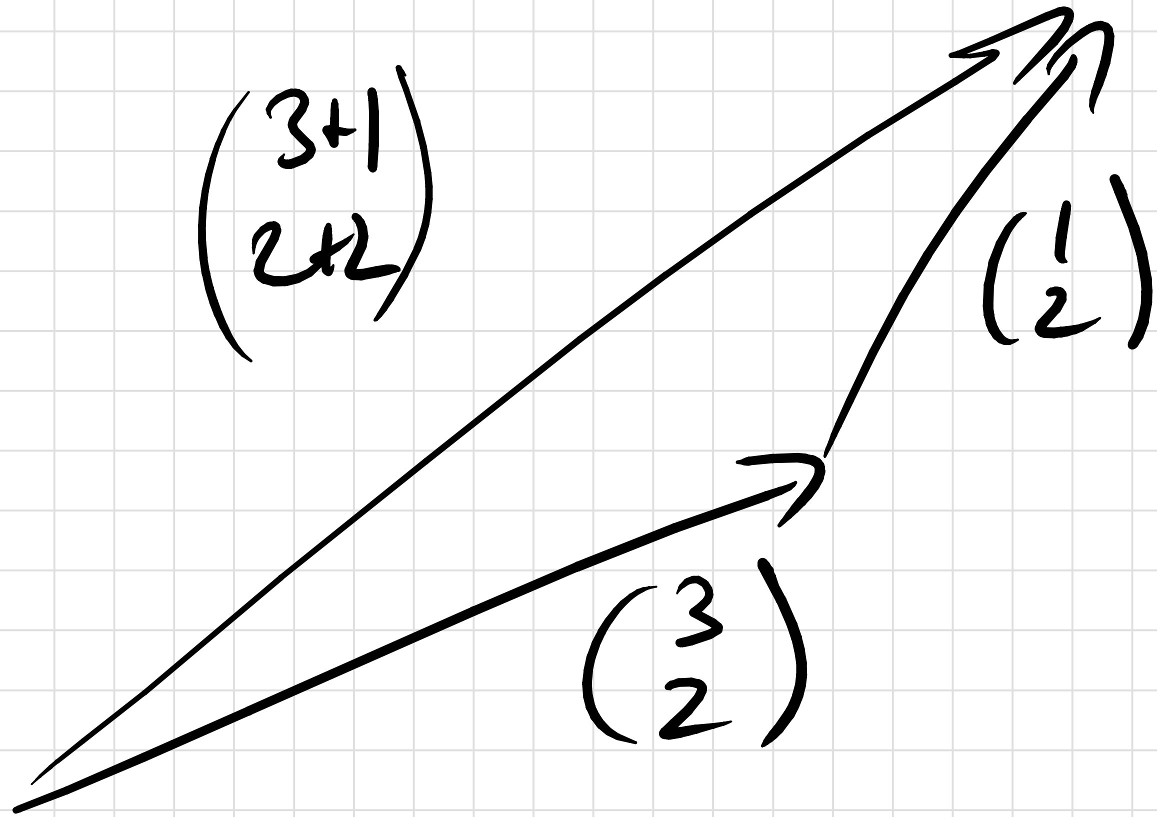
$\vec{0} = 0 \cdot \vec{b}$


zu mir hin
von mir weg

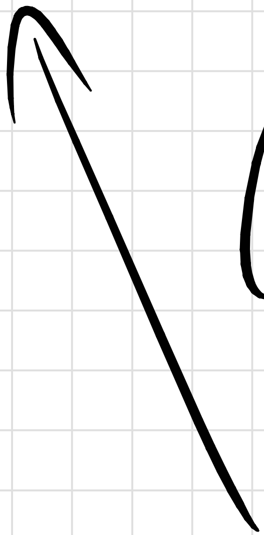

aus dem Papier heraus
ins Papier hinein



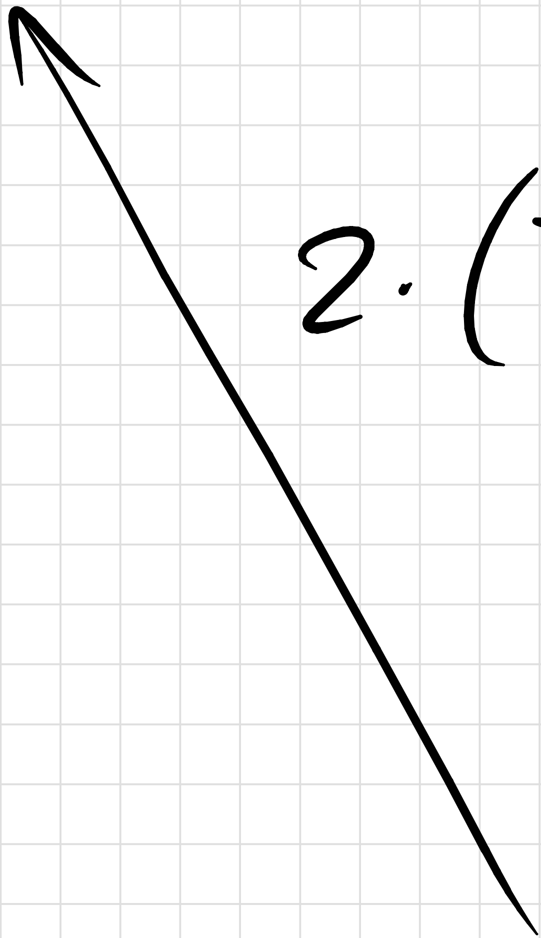
1 kartesisches Koordinatensystem


$\begin{pmatrix} 3+1 \\ 2+2 \end{pmatrix}$
$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$
$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 3+2 \\ 2+(-2) \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$



$$\begin{pmatrix}-1\\3\end{pmatrix}$$

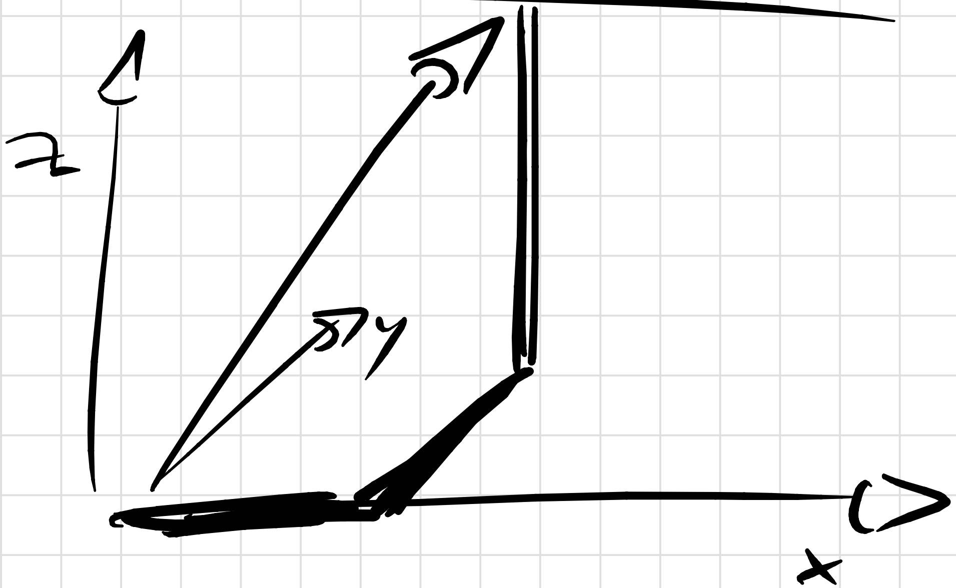


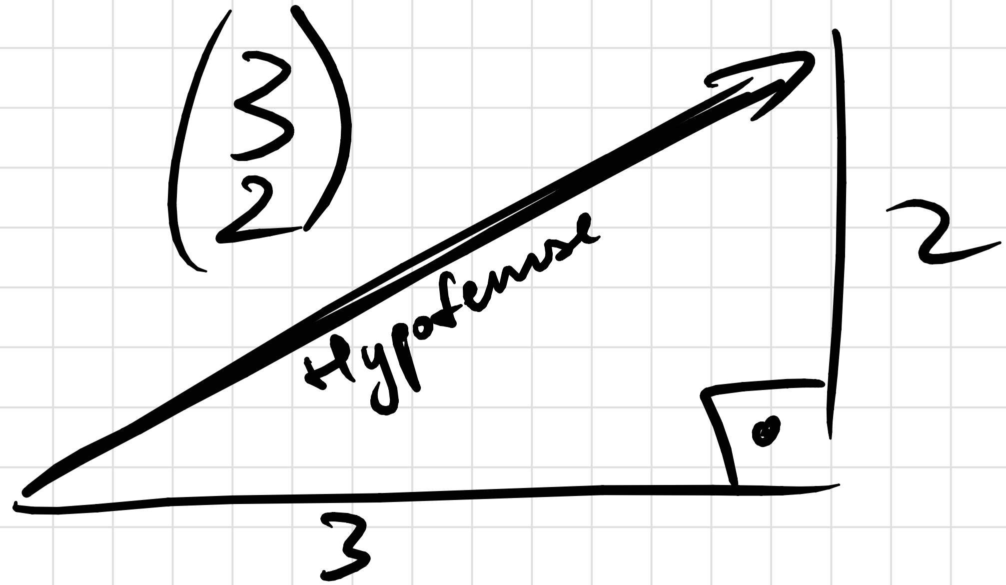
$$2\cdot\begin{pmatrix}-1\\3\end{pmatrix} = \begin{pmatrix}2\cdot(-1)\\2\cdot 3\end{pmatrix} = \begin{pmatrix}-2\\6\end{pmatrix}$$

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1-1 \\ 2-2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

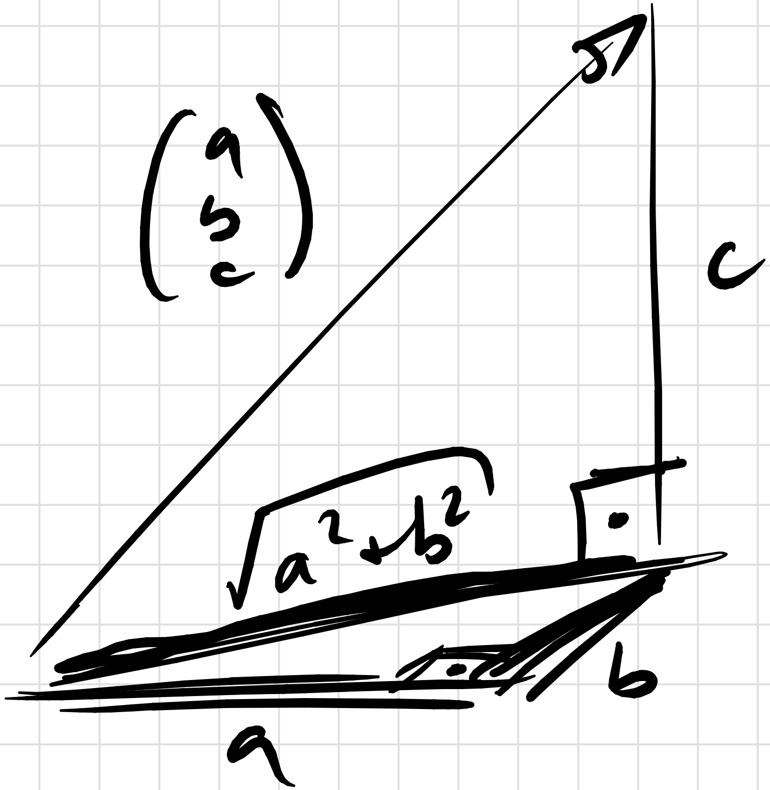
3D:

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$





$$\left\| \begin{pmatrix}3\\2\end{pmatrix} \right\| = \sqrt{3^2+2^2}$$



$$\left\| \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\| = \sqrt{\sqrt{a^2+b^2}^{\,2} + c^2}$$

$$= \sqrt{a^2+b^2+c^2}$$

Vektor + Vektor = Vektor

" — " = "

Skalar • Vektor = Vektor

$\boxed{\text{Vektor} \cdot \text{Vektor} = \text{Skalar}}$

Skalarprodukt

Vektor × Vektor = Vektor

Vektorprodukt

= Kreuzprodukt

nur in 3D !

Skalarprodukt

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \end{pmatrix} = 2 \cdot 4 + 3 \cdot (-1)$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \left(\begin{pmatrix} 4 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

Ausklammern!

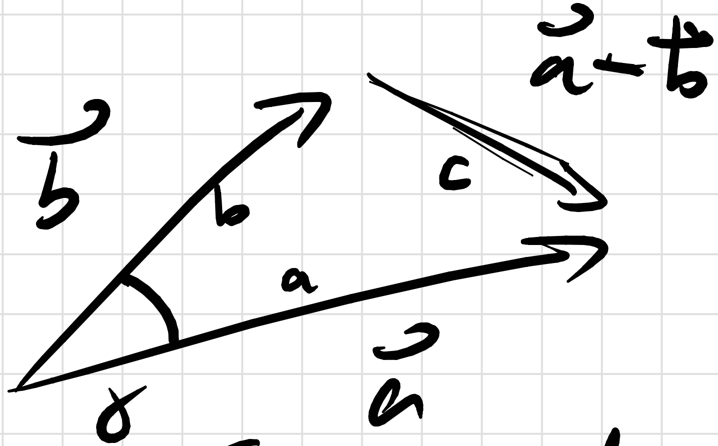
$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \left(5 \begin{pmatrix} 4 \\ -1 \end{pmatrix} \right) = 5 \cdot \left(\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \end{pmatrix} \right)$$

Skalarprodukt geometrisch:

$$\begin{pmatrix}1\\2\end{pmatrix} \cdot \begin{pmatrix}1\\2\end{pmatrix} = 1 \cdot 1 + 2 \cdot 2$$
$$= 1^2 + 2^2$$
$$= \left\| \begin{pmatrix}1\\2\end{pmatrix} \right\|^2$$

Allgemein: $\vec{a} \bullet \vec{a} = \|\vec{a}\|^2$
für alle $\vec{a}$.

$$\|\vec a - \vec b\|^2$$



Cosinussatz:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

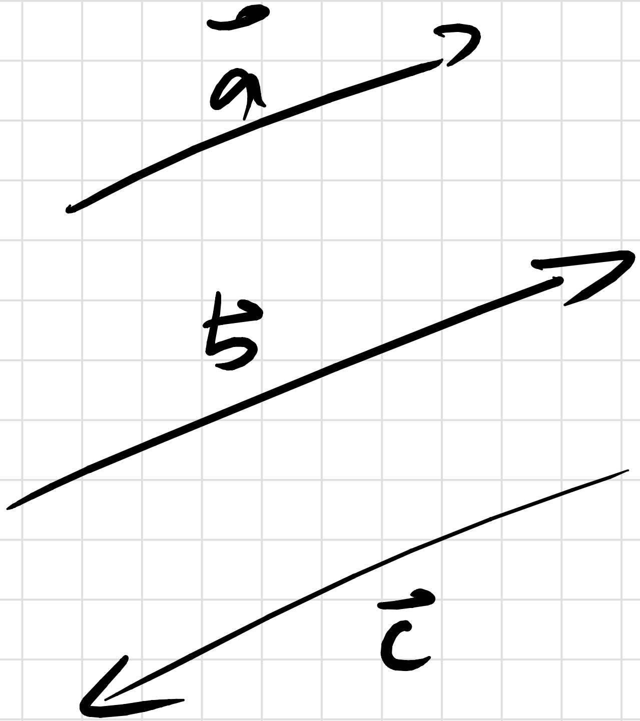
$$= (\vec a - \vec b)\cdot(\vec a - \vec b)$$

$$= (\vec a - \vec b)\cdot \vec a - (\vec a - \vec b)\cdot \vec b$$

$$= \underbrace{\vec a \cdot \vec a}_{\|\vec a\|^2} \underbrace{- \vec b\cdot\vec a - \vec a\cdot\vec b}_{\underbrace{-\vec a\cdot\vec b - \vec a\cdot\vec b}_{-2\vec a\cdot\vec b}} + \underbrace{\vec b\cdot\vec b}_{\|\vec b\|^2}$$

$$\Longrightarrow \boxed{\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos\left(\text{Winkel dazwischen}\right)}$$

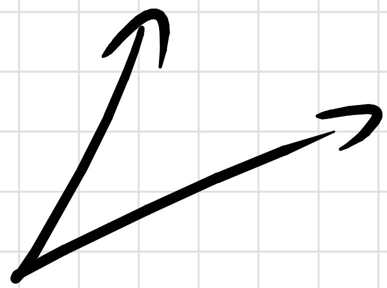
Geometrie des Skalarprodukts



$$\vec{a} \cdot \vec{a} = \|\vec{a}\|^2$$

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\|$$

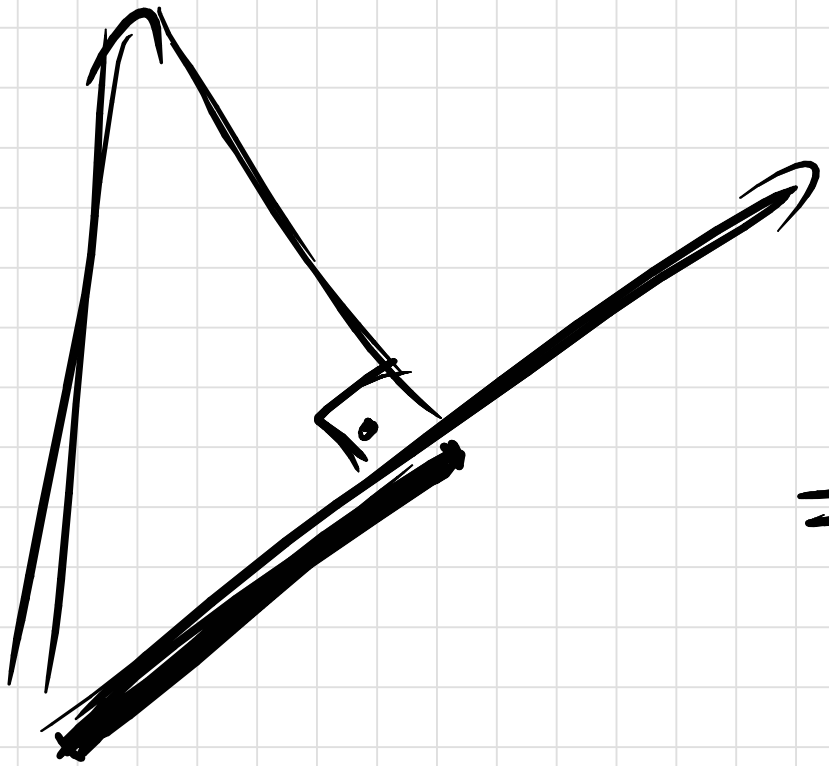
$$\vec{a} \cdot \vec{c} = -\|\vec{a}\| \cdot \|\vec{c}\|$$



Skalarprod. >0

Skalarprod. $=0$

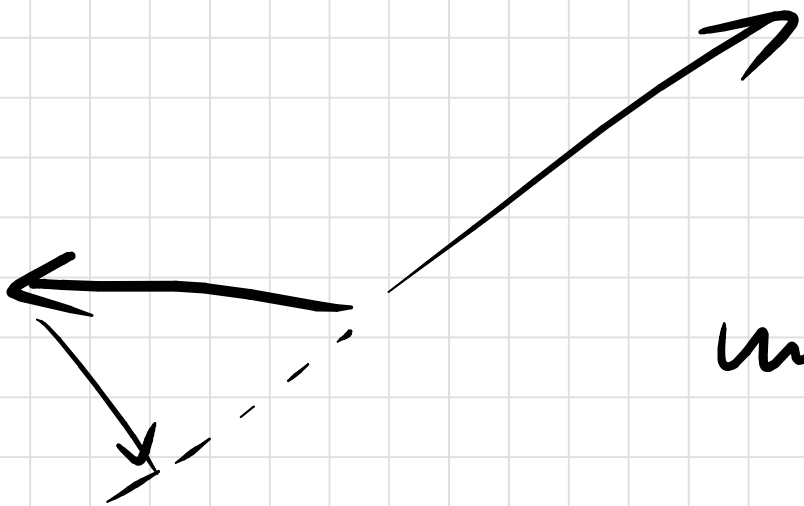
Skalarprod. <0



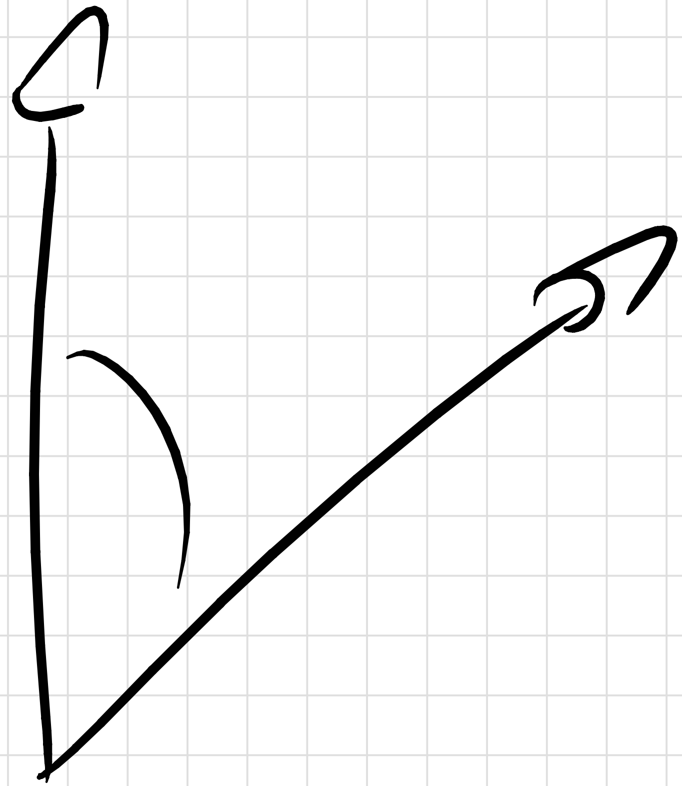
Skalarprodukt

= Länge des Schattens des einen Vektor

· den anderen Vektor



mit Vorzeichen!



Vektorprodukt nur in 3D

$$\begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \cdot 1 - 3 \cdot 5 \\ 3 \cdot 2 - 5 \cdot 1 \\ 5 \cdot 5 - 4 \cdot 2 \end{pmatrix}$$

$$= \begin{pmatrix} -11 \\ 1 \\ 17 \end{pmatrix}$$

senkrecht zu beiden Faktoren!

Denn:
$$\begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -11 \\ 1 \\ 17 \end{pmatrix} = -55 + 4 + 51 = 0$$

$$\begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -11 \\ 1 \\ 17 \end{pmatrix} = -22 + 5 + 17 = 0$$

✓

$$(\lambda \vec{a}) \times \vec{b} = \lambda \, (\vec{a} \times \vec{b})$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \times \vec{a} = -\vec{a} \times \vec{a}$$

$$\Rightarrow \vec{a} \times \vec{a} = \vec{0}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\overline{\|\vec{a} \times \vec{b}\|^2 = \|\vec{a}\|^2 \cdot \|\vec{b}\|^2 - (\vec{a} \cdot \vec{b})^2}$$

$$= \|\vec{a}\|^2 \cdot \|\vec{b}\|^2 - \left(\|\vec{a}\| \cdot \|\vec{b}\| \cos(\ldots)\right)^2$$

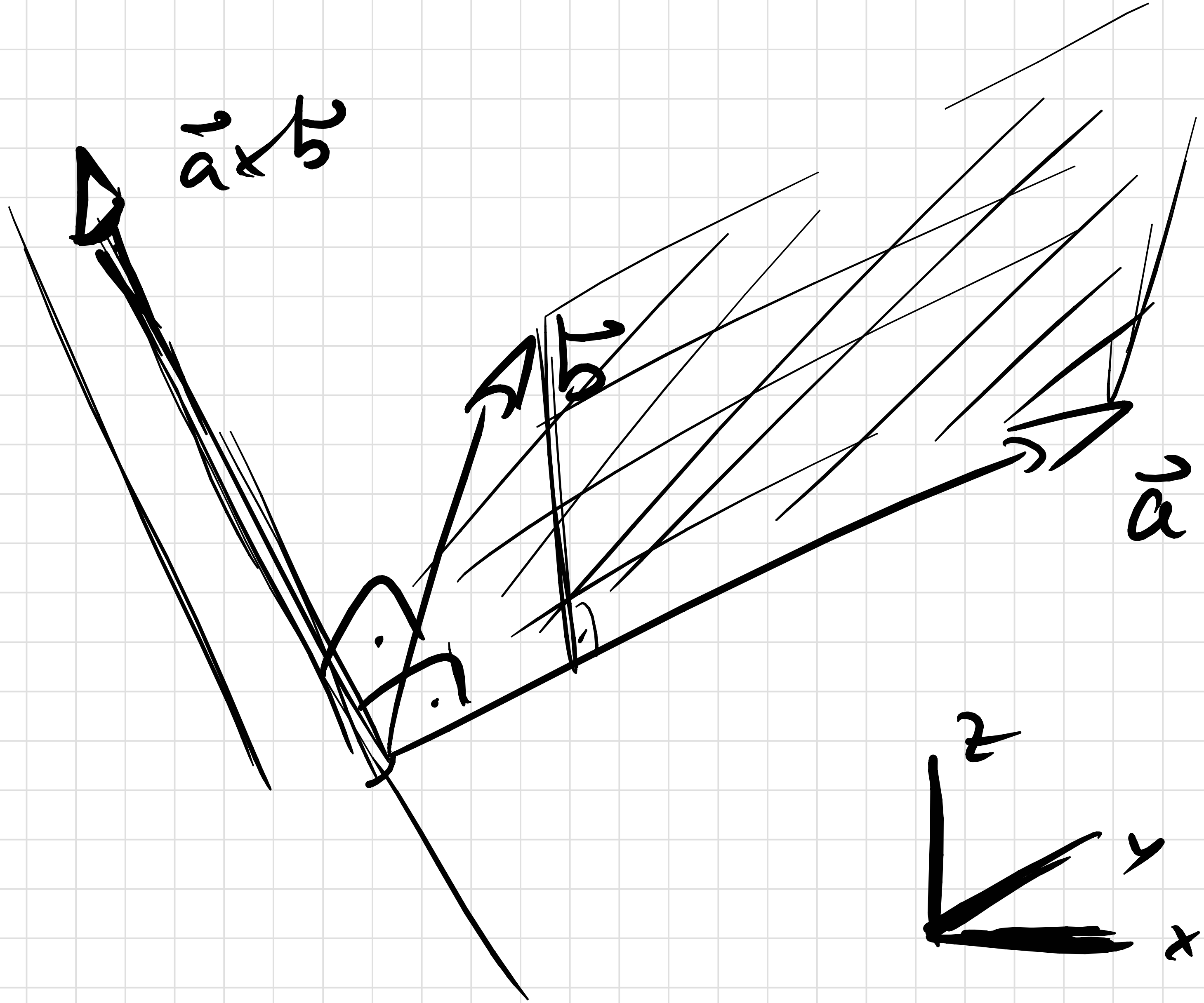
$$= \|\vec{a}\|^2 \cdot \|\vec{b}\|^2 - \|\vec{a}\|^2 \cdot \|\vec{b}\|^2 \cdot \cos^2(\ldots)$$

$$= \|\vec{a}\|^2 \cdot \|\vec{b}\|^2 \cdot \Big(\underbrace{1 - \cos^2(\ldots)}_{\sin^2(\ldots)}\Big)$$

$$\Longrightarrow$$

$$\boxed{\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \left|\sin(\ldots)\right|}$$

$$\sqrt{(-3)^2} = 3$$


$\vec{a} \times \vec{b}$
$\vec{b}$
$\vec{a}$
z
y
x