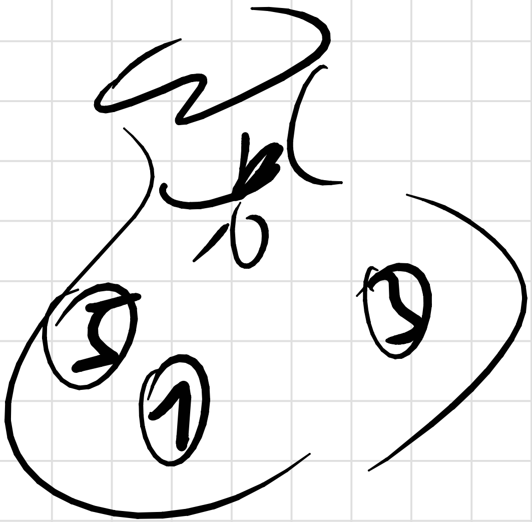
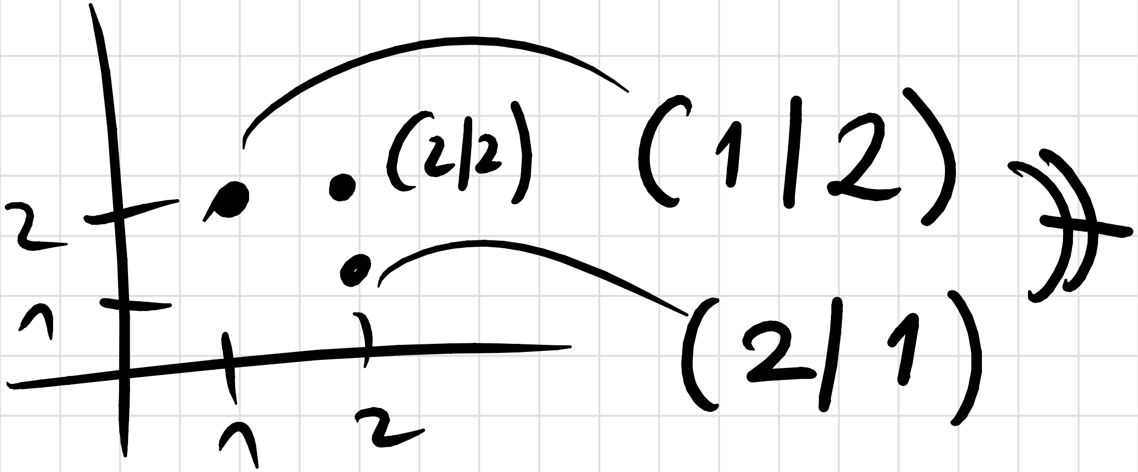
„Naive" Mengenlehre



$$= \{1, 3, 5\} = \{5, 1, 3\}$$

$$\cancel{\{5, 5, 3\}}$$

Elemente



(2|2)
(1|2)
(2|1)
2
1
1
2

$$5 \in \{1, 3, 5\}$$

$$6 \notin \{1, 3, 5\}$$

$\{\} = \emptyset$ leere Menge

Mächtigkeit = Anzahl der Elemente

$$\# \{1, 3, 5, 9\}$$

$$= |\{1, 3, 5, 9\}|$$

$$= 4$$

endliche Menge

$$\# \{1, 2, 3, 4, \ldots \}$$

$$= \infty$$

unendliche Menge

echte Teilmenge echte Obermenge

$$\{2, 3, 5\} \subset \{1, 2, 3, 4, 5\}$$

Teilmenge Obermenge

$$\{1, 2, 3, 4, 5\} \subseteq \{1, 2, 3, 4, 5\}$$

$\{1, 2, 3\} \cup \{2, 4, 6, 7\}$

$\cap$

Vereinigungsmenge

Schnittmenge

$= \{1, 2, 3, 4, 6, 7\}$

$\{2\}$

Mengendifferenz

$$\{1, 2, 3\} \setminus \{2, 4, 6, 7\}$$

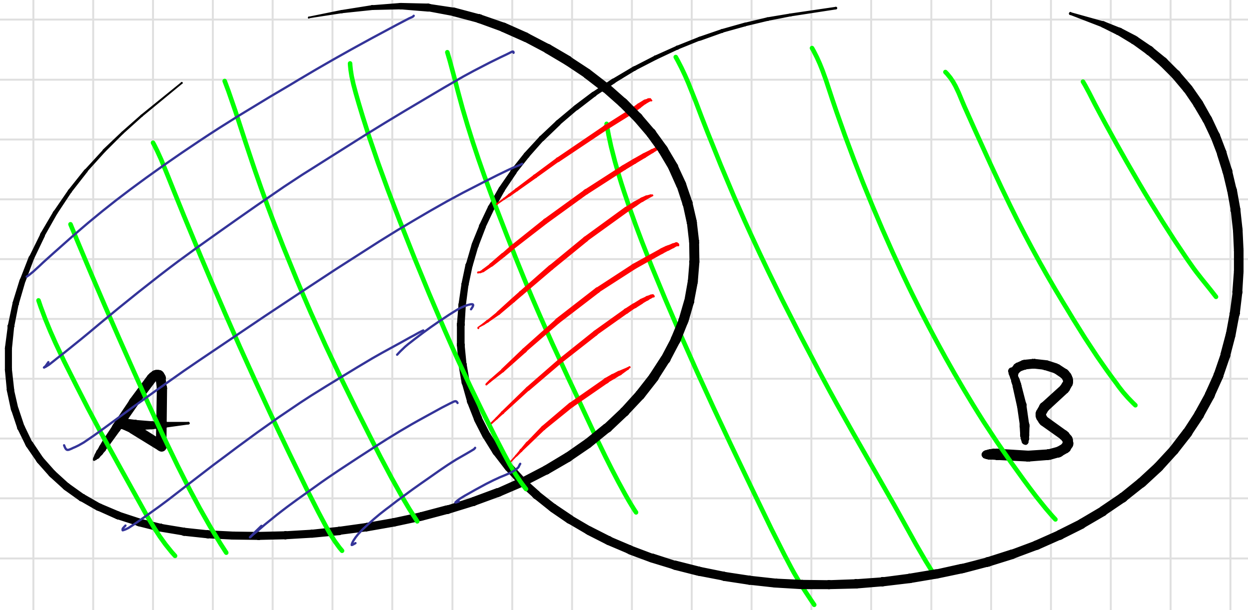
$$= \{1, 3\}$$

Komplement

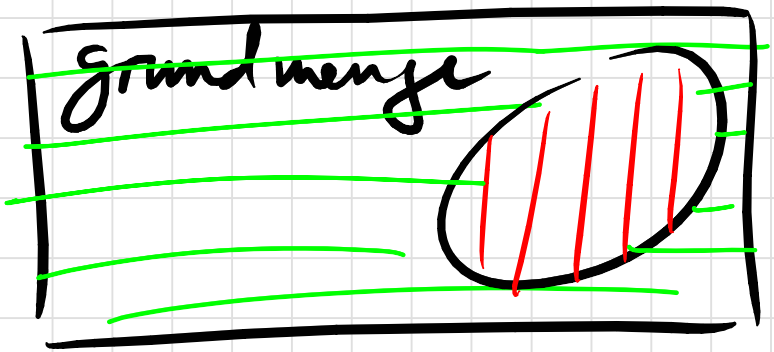
bezüglich einer Grundmenge

z. B. Grundmenge = $\{1, 2, 3, 4, \dots\}$

$$\overline{\{3, 5, 9\}} = \{1, 2, 4, 6, 7, 8, 10, \dots\}$$



$A \cap B$ $A \cup B$ $A \setminus B$



A $\bar{A}$

$$A \cup B = B \cup A$$

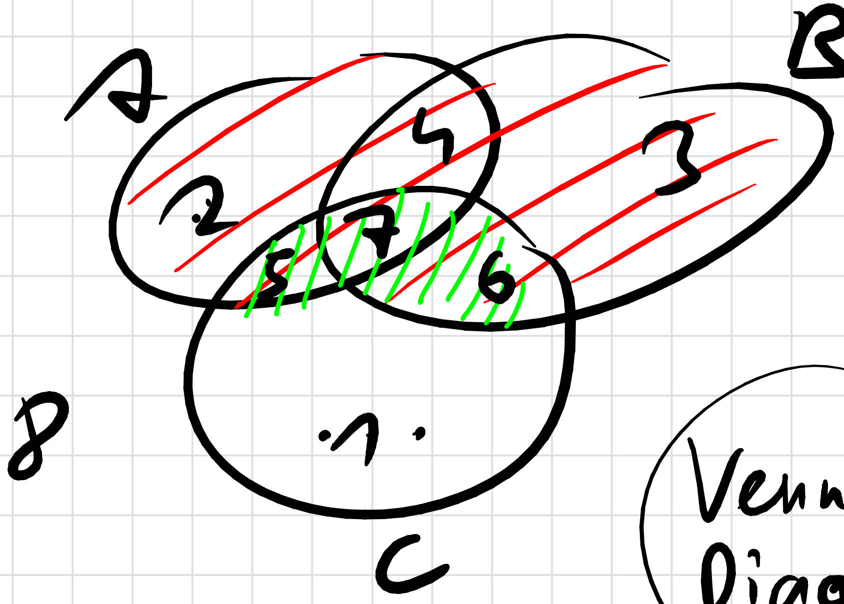
$$A \cap B = B \cap A$$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

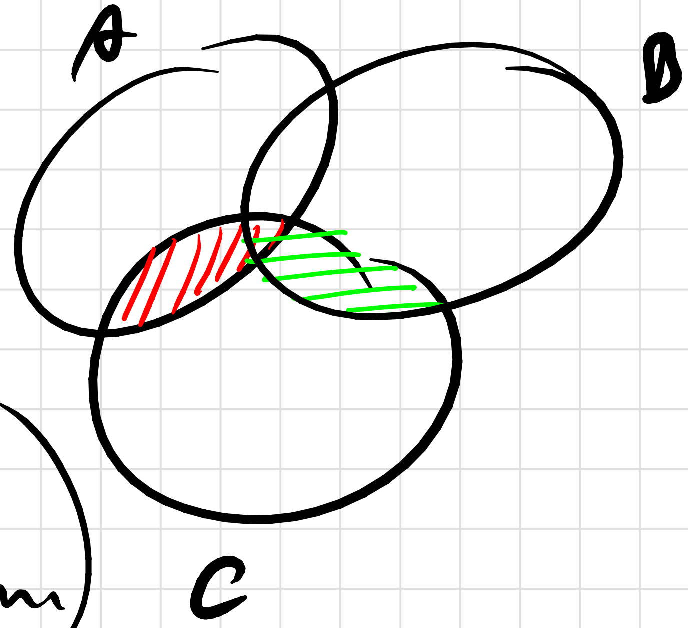
$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$(A \cup B) \cap C = A \cap C \cup B \cap C$$

1. 2. 1.



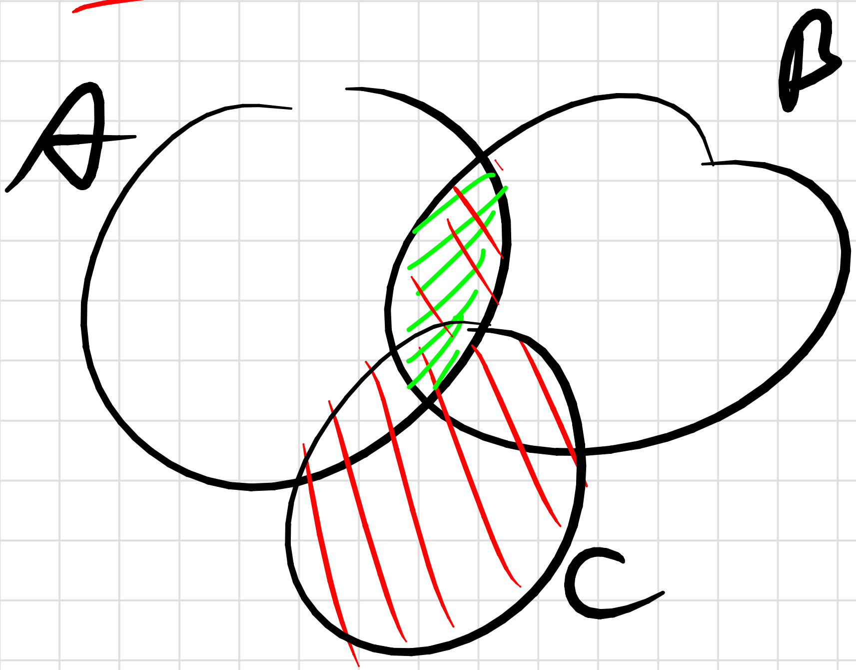
Venn-Diagramm

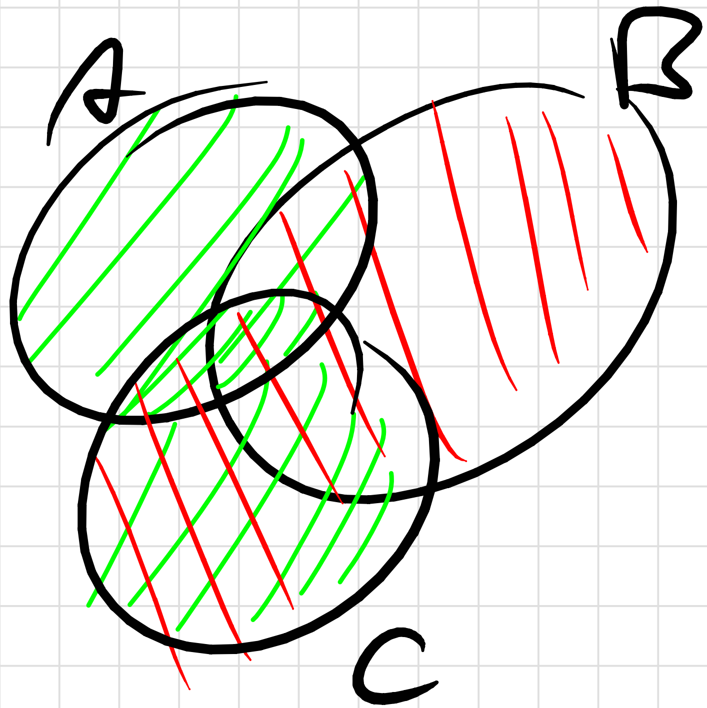


$$(3+4)\cdot 5 = 3\cdot 5+4\cdot 5$$

$$\cancel{(3\cdot 4)+5 = (3+5)\cdot(4+5)}$$

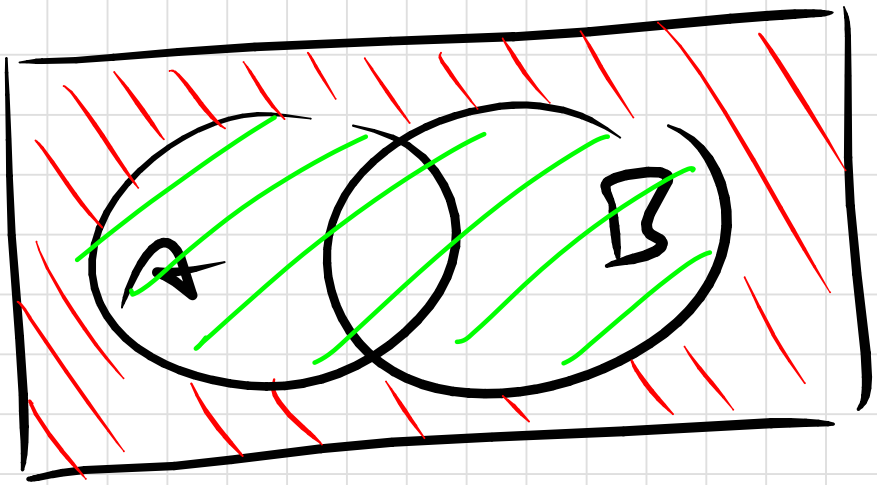
$$A \cap B \cup C = (A \cup C) \cap (B \cup C)$$

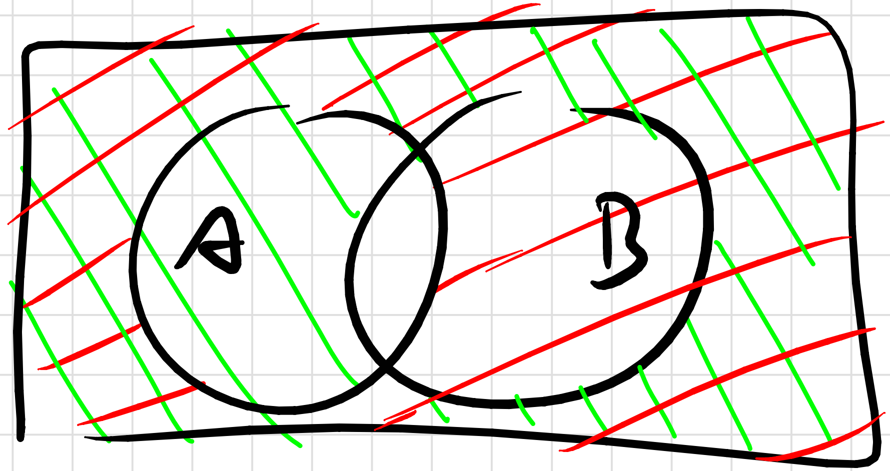




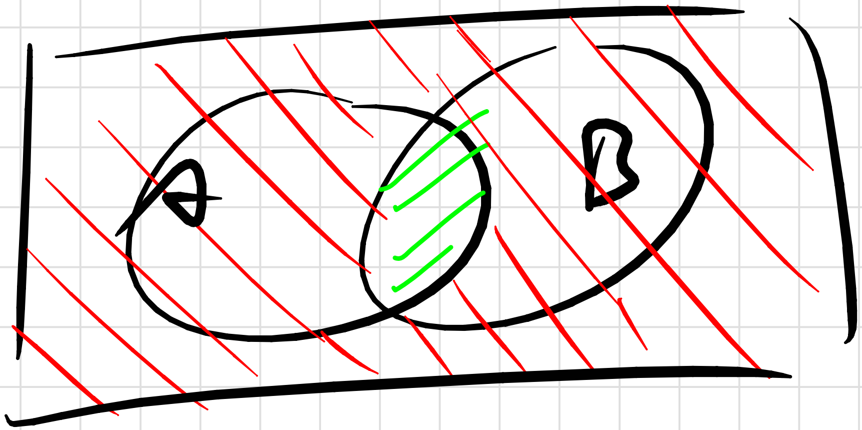
De Morgan

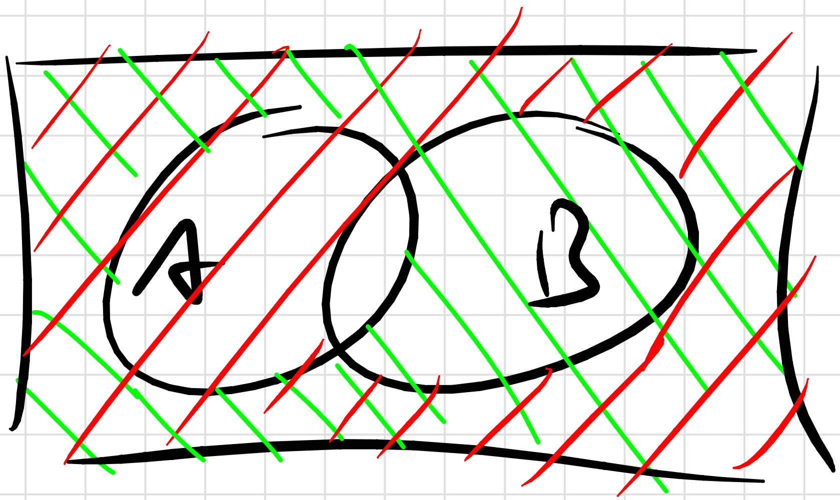
$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

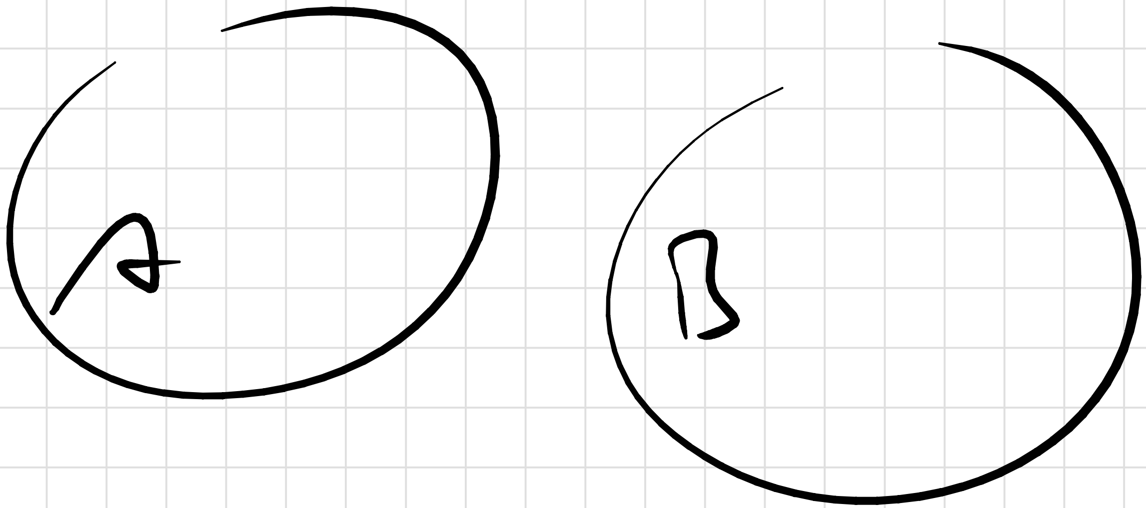




$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$







$$A \cap B = \emptyset$$

„A und B sind disjunkt."

$$3 \in \{1, 2, 3\}$$

$$2 \in \{2\}$$

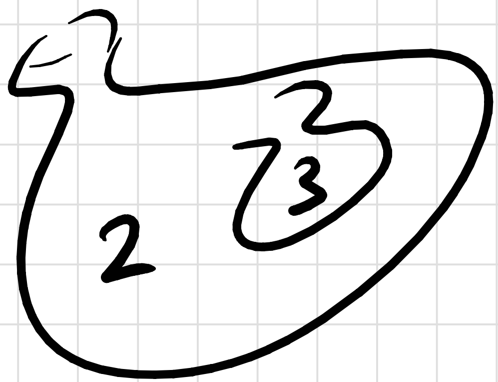
$$\cancel{2 = \{2\}}$$

$$\cancel{\{1,2,3\} \setminus 3}$$

$$\{1,2,3\} \setminus \{3\}$$

$$\{2, \{3\}\} \neq \{\{2\}, 3\}$$

$$\neq \{2, 3\}$$



$$\underset{0}{\{\}}, \quad \underset{1}{\{\{\}\}}, \quad \underset{2}{\{\{\},\{\{\}\}\}}, \ldots$$

Zahlenbereiche

$\mathbb{N}$ natürliche Zahlen

$$\mathbb{N}_0 = \{0, 1, 2, 3, \ldots\}$$

$$\mathbb{N}^+ = \mathbb{Z}^+ = \{1, 2, 3, \ldots\}$$

$\mathbb{Z}$ ganze Zahlen

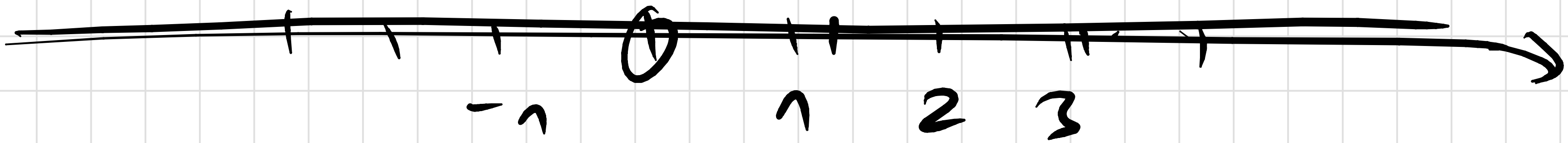
$$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$$

$\mathbb{Q}$ rationale Zahlen

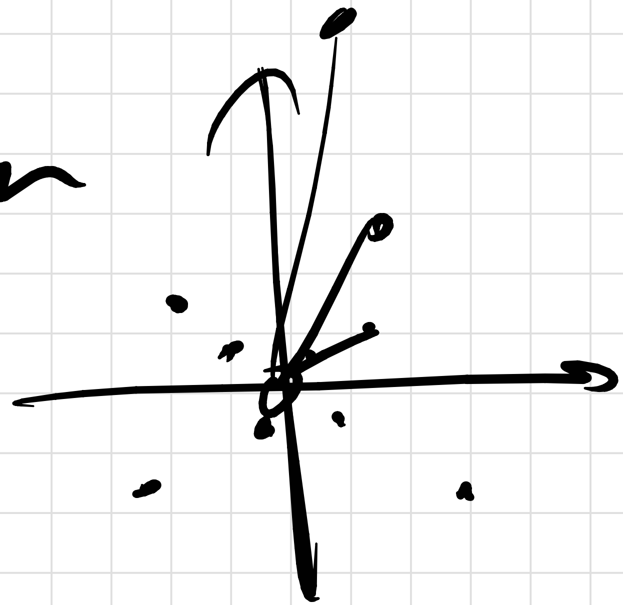
$$\left\{\frac{1}{3}, \frac{7}{400}, -\frac{93}{80}, 5, 42, 0, \ldots\right\}$$

$\mathbb{R}$ reelle Zahlen

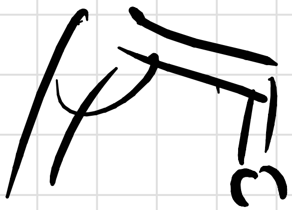
$\{\sqrt{2}, \pi, \frac{4}{301}, -98, 0, \dots\}$



$\mathbb{C}$ komplexe Zahlen



$\mathbb{H}$ Quaternionen



Menge aller reellen Zahlen ab π aufwärts

$$= \{3{,}15;\ 4;\ 42;\ 98{,}778 \ldots\}$$

$$= \{x \in \mathbb{R} : x \geq \pi\}$$

Auswahl durch ein Prädikat

$$\cancel{\{x : 3x = 4\}}$$

$$\{x \in \mathbb{Q} : 3x = 4\} = \left\{\frac{4}{3}\right\}$$

$$\{\underline{x \in \mathbb{Z}} : 3x = 4\} = \emptyset$$

$$\Rightarrow \cancel{\{x : \underline{x \in \mathbb{Z}} \wedge 3x = 4\}}$$

und

$x = 2 \land x \geq 5$

und

$x = 2 \lor x \geq 5$

nicht

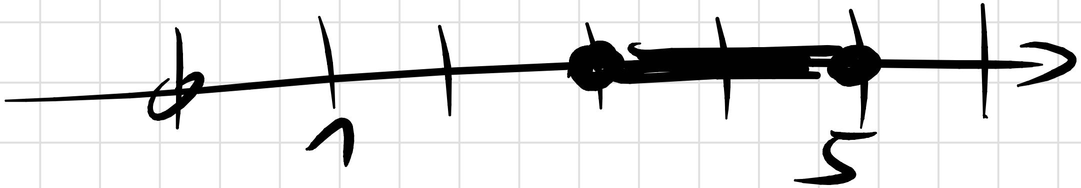
oder

$\neg (x = 2)$

Intervalle reeller Zahlen

abgeschlossen

$$[3;5] = \{3;\ \pi;\ 4;\ 4{,}2;\ 3{,}298;\ 5;\ \dots\}$$

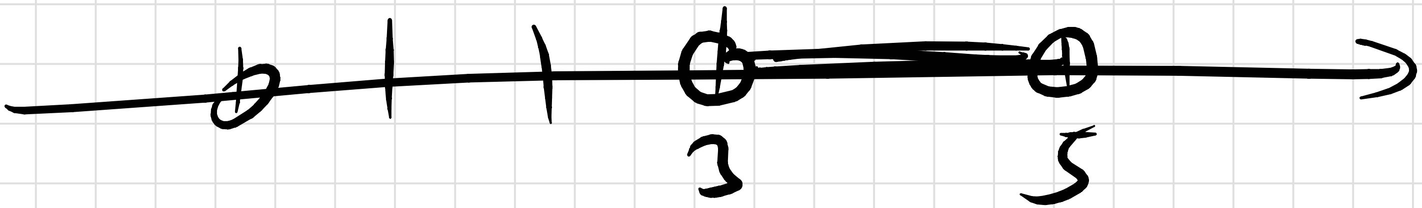


$$= \{x \in \mathbb{R} : x \geq 3 \wedge x \leq 5\}$$

often

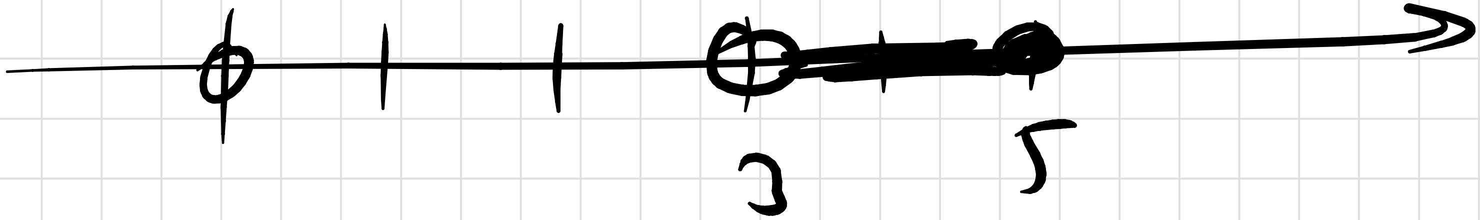
$$(3;5) = \,]3;5[$$

$$= \{x \in \mathbb{R} : x > 3 \wedge x < 5\}$$

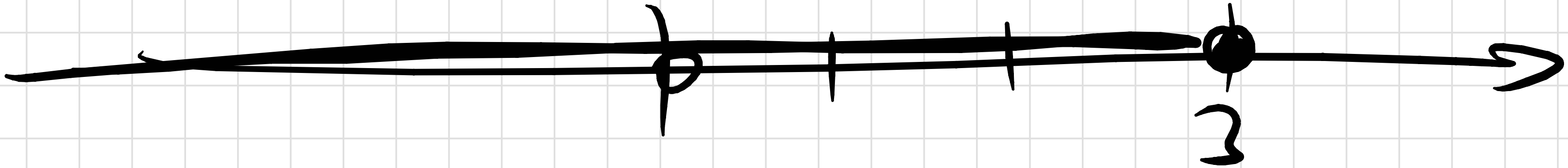


$$(3;5] = \,]3;5]$$

$$= \{x \in \mathbb{R} : x > 3 \wedge x \leq 5\}$$



$$(-\infty; 3] = \{x \in \mathbb{R} : x \leq 3\}$$



$$(-\infty; \infty) = \mathbb{R}$$

$$[0; \infty) = \mathbb{R}_0^+$$

$$(0; \infty) = \mathbb{R}^+$$

Ungleichungen

$$3x + 4 > 5$$

$$\mathbb{L} = \{ x \in \mathbb{R} : 3x + 4 > 5 \}$$

$$3x + 4 > 5 \qquad \| -4$$

$$\Leftrightarrow 3x + \cancel{4} - \cancel{4} > \underbrace{5 - 4}_{1}$$

genau dann wenn

$$\Leftrightarrow 3x > 1 \qquad \| :3$$

$$\Leftrightarrow \quad x > 1/3$$

$$\mathbb{L} = \{x \in \mathbb{R} : x > 1/3\}$$
$$= (1/3 ; \infty)$$

$$|3x+4| > 5$$

$$\iff 3x+4 \geq 0 \wedge |3x+4| > 5$$

$$\vee\ 3x+4 < 0 \wedge |3x+4| > 5$$

$$\iff \boxed{3x+4 \geq 0} \wedge \boxed{3x+4 > 5}$$

$$\vee\ 3x+4 < 0 \wedge -(3x+4) > 5$$

$$\Leftrightarrow \quad 3x+4>5$$

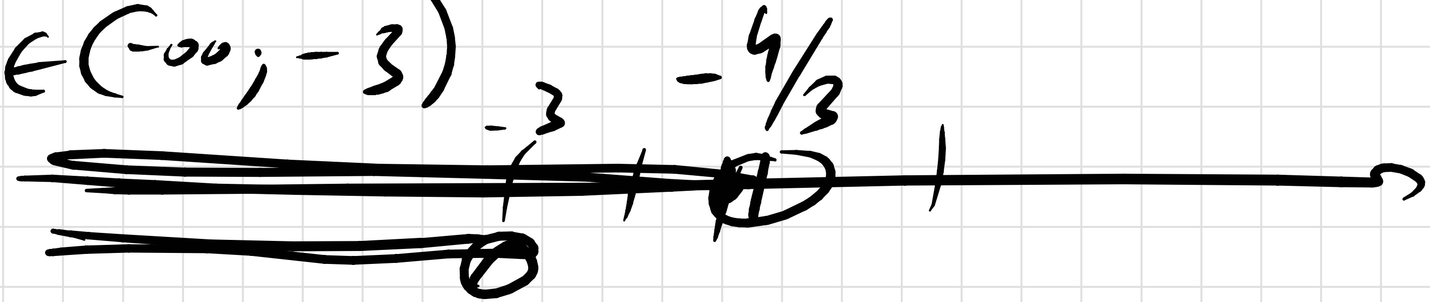
$$\vee \quad 3x+4<0 \quad \wedge \quad 3x+4<-5$$

$$\Leftrightarrow \quad x \in \left(\tfrac{1}{3};\infty\right)$$

$$\vee \quad x<-\tfrac{4}{3} \quad \wedge \quad x<-3$$

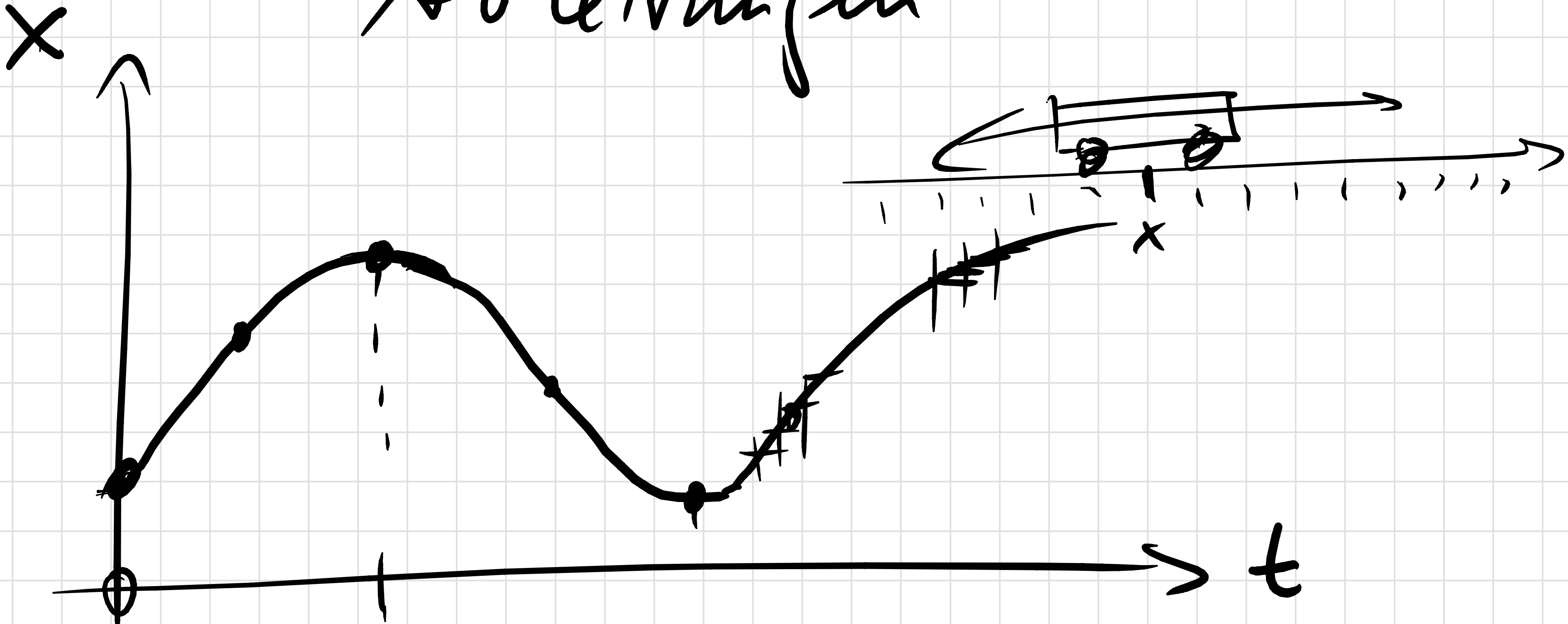
$$\Leftrightarrow \quad x \in \left(\tfrac{1}{3};\infty\right)$$

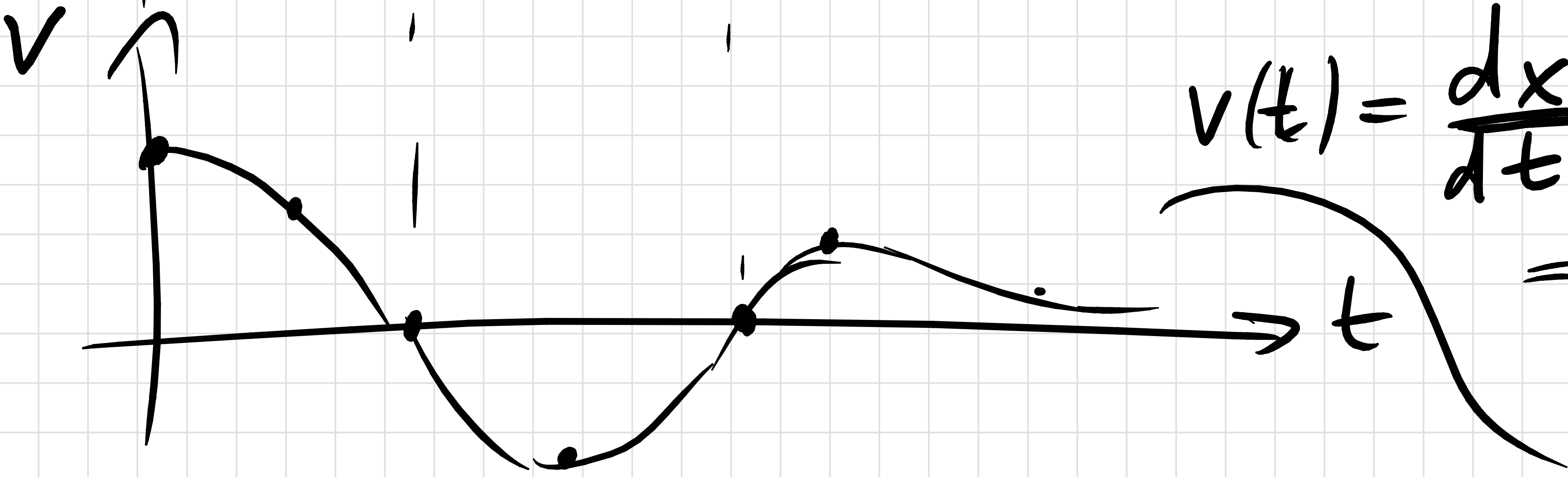
$$\vee \quad x \in (-\infty;-3)$$



$$\mathbb{L} = (-\infty ; -3) \cup (1/3 ; \infty)$$

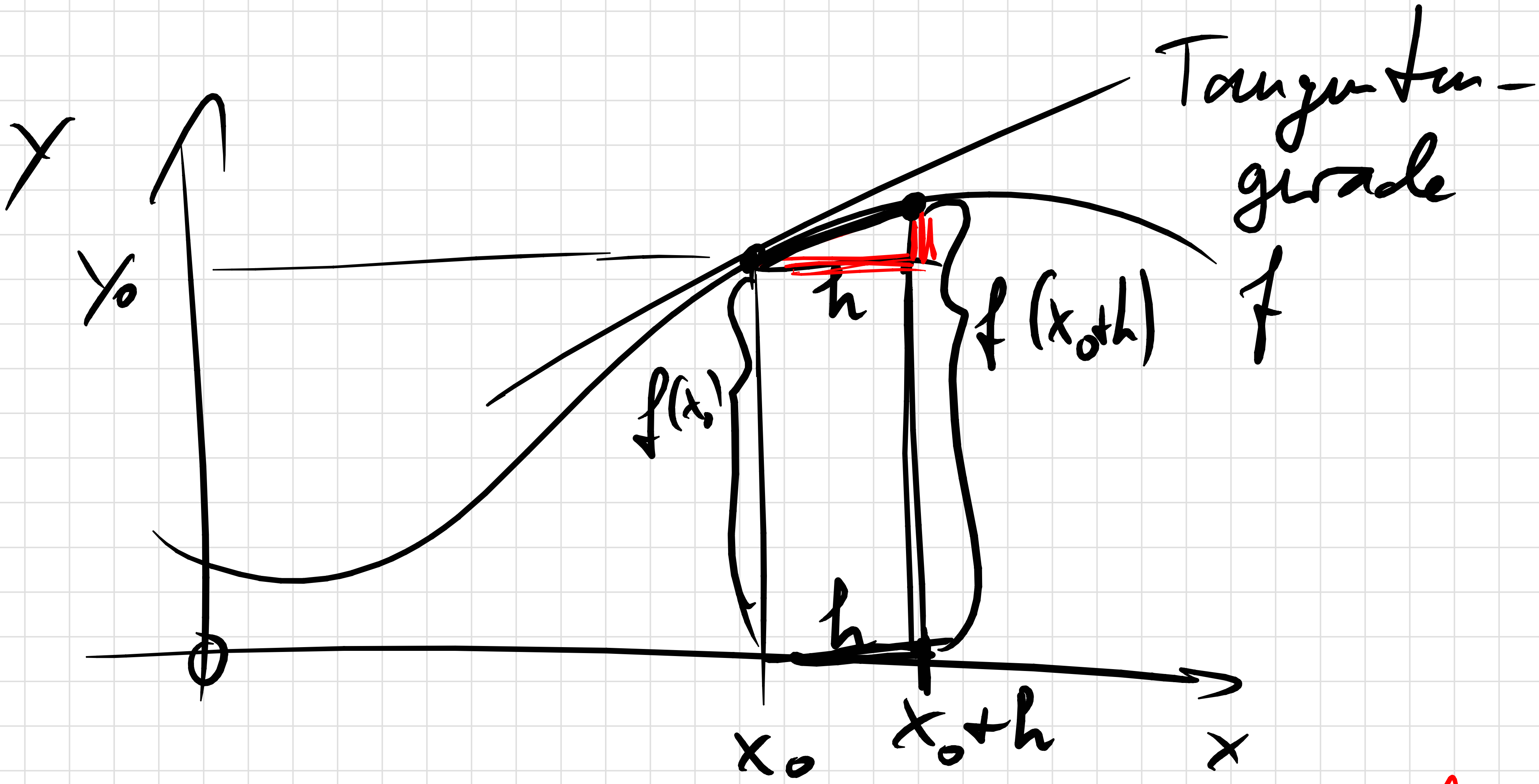
Ableitungen





$$v(t) = \frac{dx(t)}{dt}$$

$$= \dot{x}(t)$$



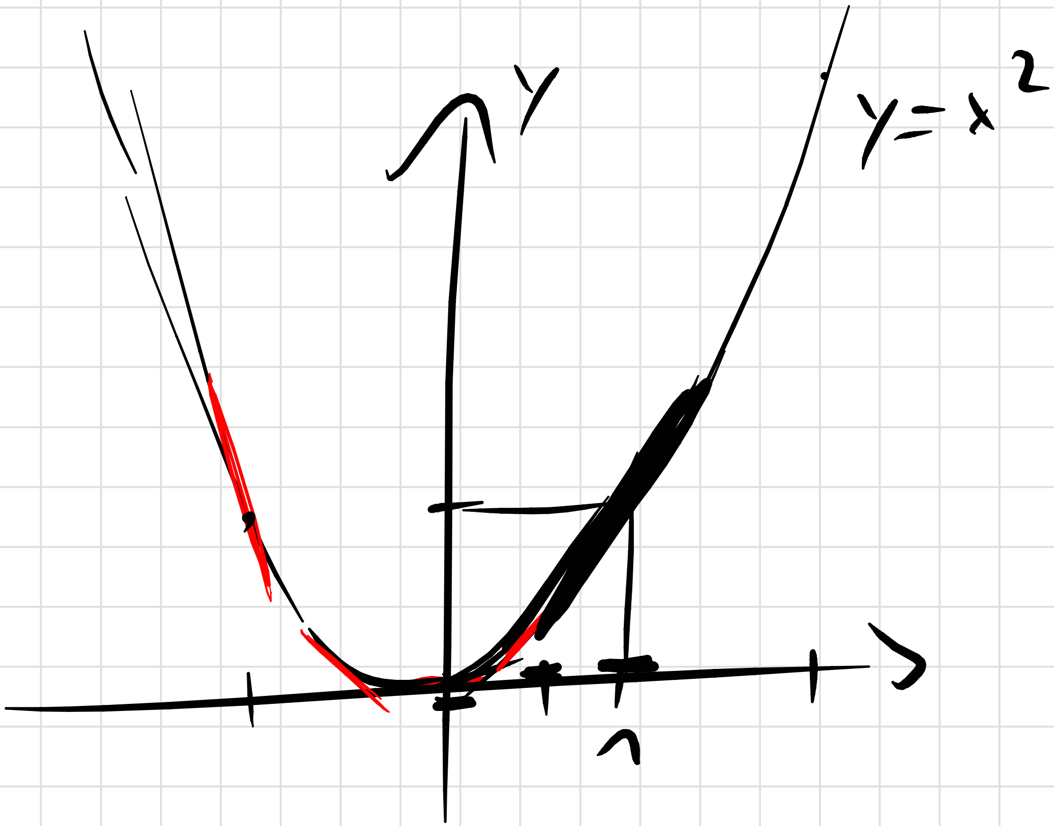
Steigung der Tangente $\approx \dfrac{f(x_0+h) - f(x_0)}{h}$

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

Ableitung von $y = x^2$

$$\frac{d\,x^2}{dx} = \lim_{h \to 0} \underbrace{\frac{(x+h)^2 - x^2}{h}}_{\underbrace{\frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}}_{2x + h}}$$

$$= \lim_{h \to 0} \left(2x + h \right) = 2x$$



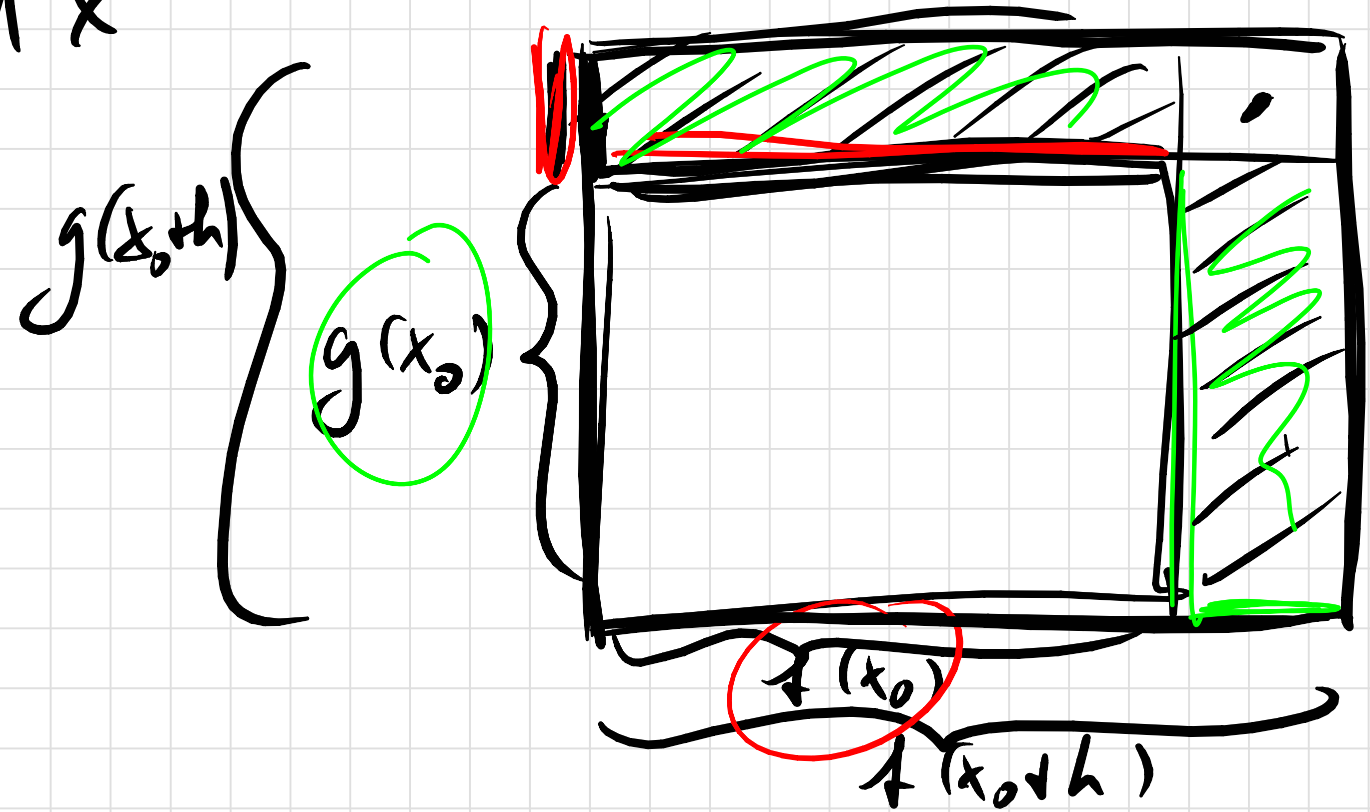
$$\frac{d\left(x^2 \pm 3x\right)}{dx} = \underbrace{\frac{dx^2}{dx}}_{2x} \pm \underbrace{\frac{d3x}{dx}}_{3}$$

$$\frac{d\,42x^2}{dx} = 42\,\frac{dx^2}{dx} = 42 \cdot 2x$$

$$= 84x$$

Produktregel

$$\frac{d\, f(x)\cdot g(x)}{dx} = f(x)\frac{dg(x)}{dx} + \frac{df(x)}{dx}g(x)$$



$$\frac{d\, x^2 \cdot x}{dx} = x^2 \frac{dx}{dx} + \frac{dx^2}{dx} x$$

$$= x^2 \cdot 1 + 2x \cdot x$$

$$= 3x^2$$

Kettenregel

$$\frac{d\, f(g(x))}{dx} = f'(g(x)) \cdot g'(x)$$

äußere · innere

Ableitung

$$\frac{d\,(\underbrace{x^2+3}^{g(x)})^2}{dx} = 2\cdot(x^2+3)\cdot 2x$$

$x > 0$

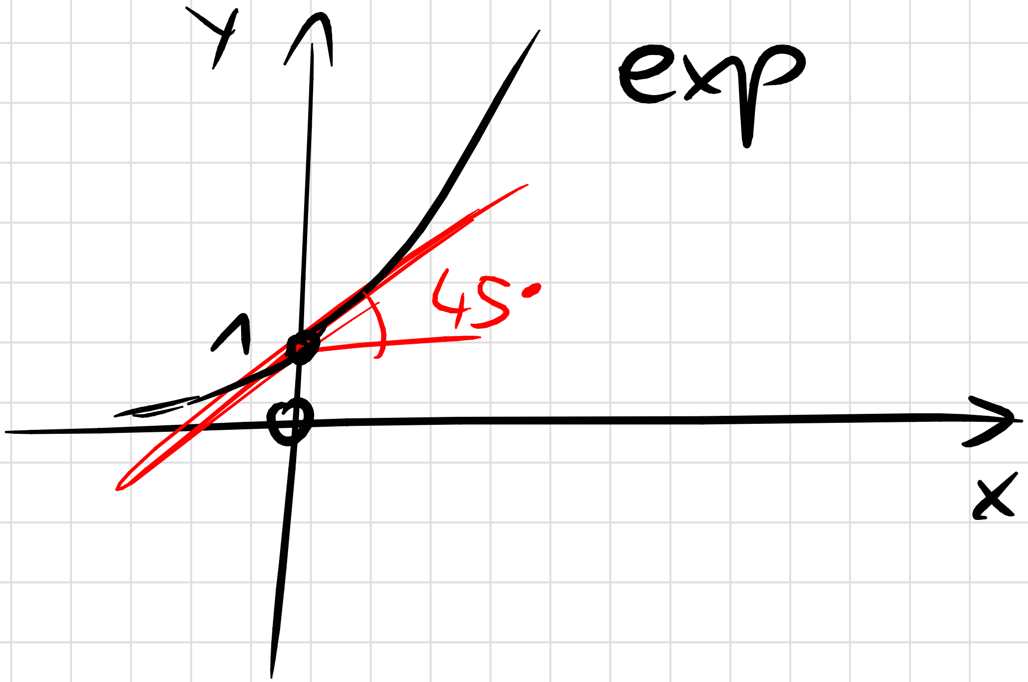
$$1 = \frac{dx}{dx} = \frac{d\,\sqrt{x}^{\,2}}{dx}$$

$$= 2\sqrt{x} \cdot \frac{d\sqrt{x}}{dx}$$

äußere innere Ableitung

$$\Longrightarrow \frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}$$

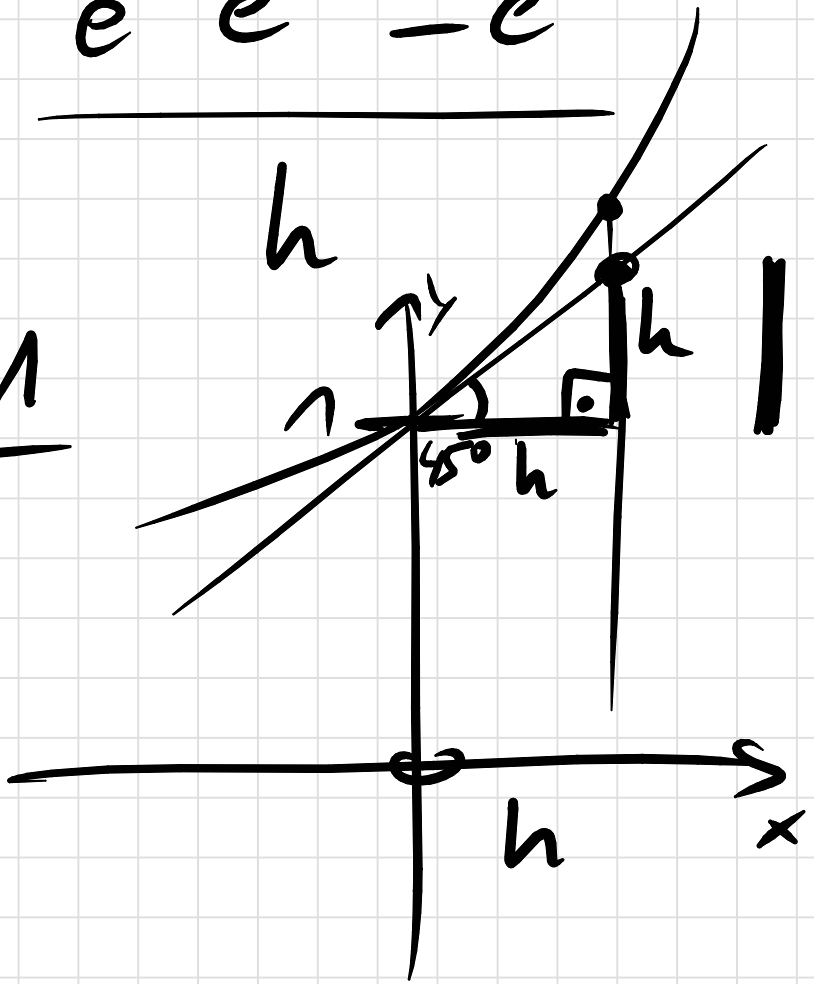
$$e^x = \exp(x)$$



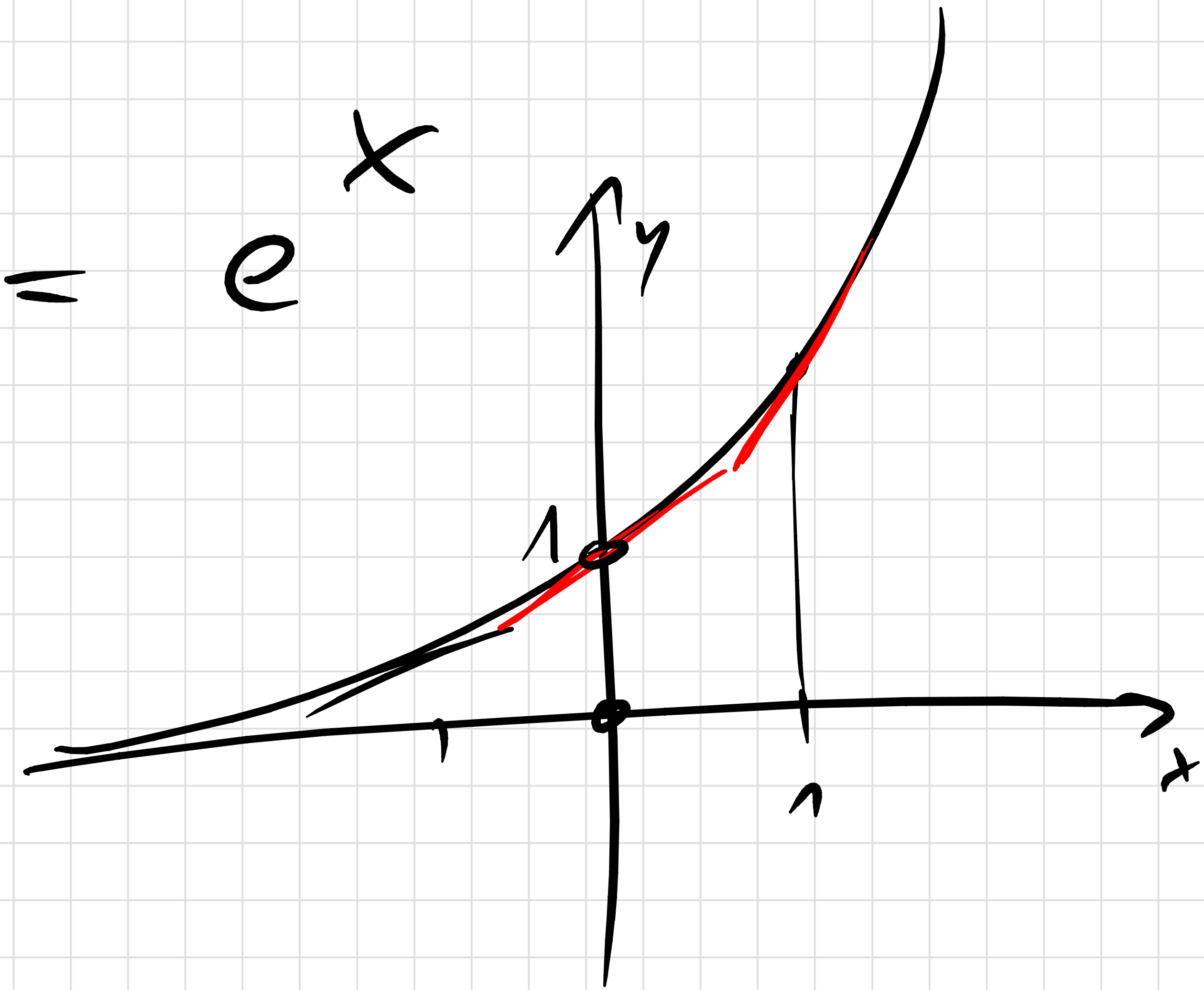
$$\frac{e^{x+h}-e^{x}}{h} = \frac{e^{x}e^{h}-e^{x}}{h}$$

$$= e^{x}\,\frac{e^{h}-1}{h}$$

$$\approx e^{x}\,\frac{\cancel{h}}{\cancel{h}}$$



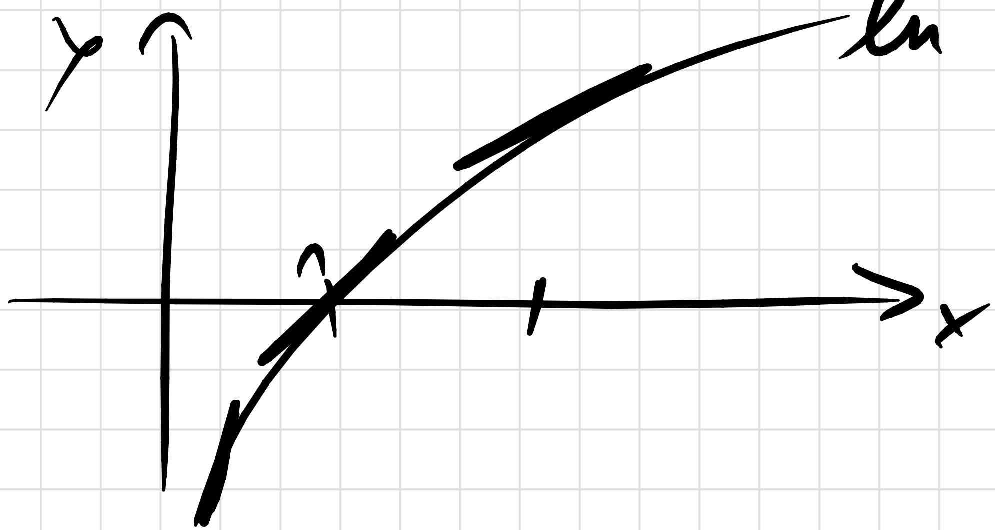
$$\frac{d\, e^x}{dx} = e^x$$



$\boxed{x > 0}$

$$1 = \frac{dx}{dx} = \frac{d\, e^{\ln(x)}}{dx} = \underbrace{e^{\ln(x)}}_{x} \cdot \frac{d\ln(x)}{dx}$$

$$\Rightarrow \quad \frac{d\ln(x)}{dx} = \frac{1}{x}$$



$$\frac{d \frac{f(x)}{g(x)}}{dx} = \frac{d\, f(x) \cdot \frac{1}{g(x)}}{dx}$$

$$= \frac{d\, f(x)}{dx} \cdot \frac{1}{g(x)} + f(x) \cdot \frac{d \frac{1}{g(x)}}{dx}$$

$$\boxed{\frac{d\frac{1}{x}}{dx} = -\frac{1}{x^2}}$$

$$-\frac{1}{g(x)^2} \cdot \frac{d\,g(x)}{dx}$$

$$= \frac{d f(x)}{dx} \cdot \frac{1}{g(x)} - \frac{f(x)}{g(x)^2} \cdot \frac{d\,g(x)}{dx}$$

$$= \frac{\frac{d f(x)}{dx} \cdot g(x) - f(x) \cdot \frac{d g(x)}{dx}}{g(x)^2}$$

$$\boxed{\frac{d\,x^{n}}{dx} = n \cdot x^{n-1}}$$