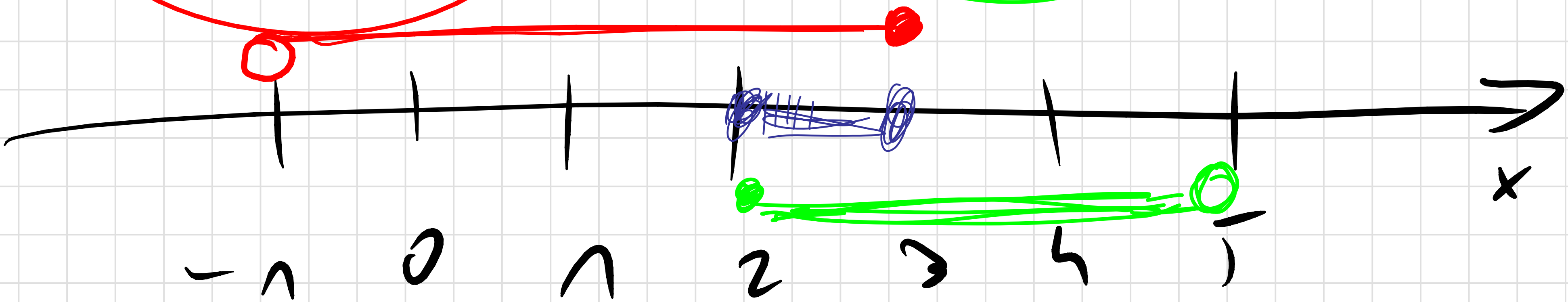
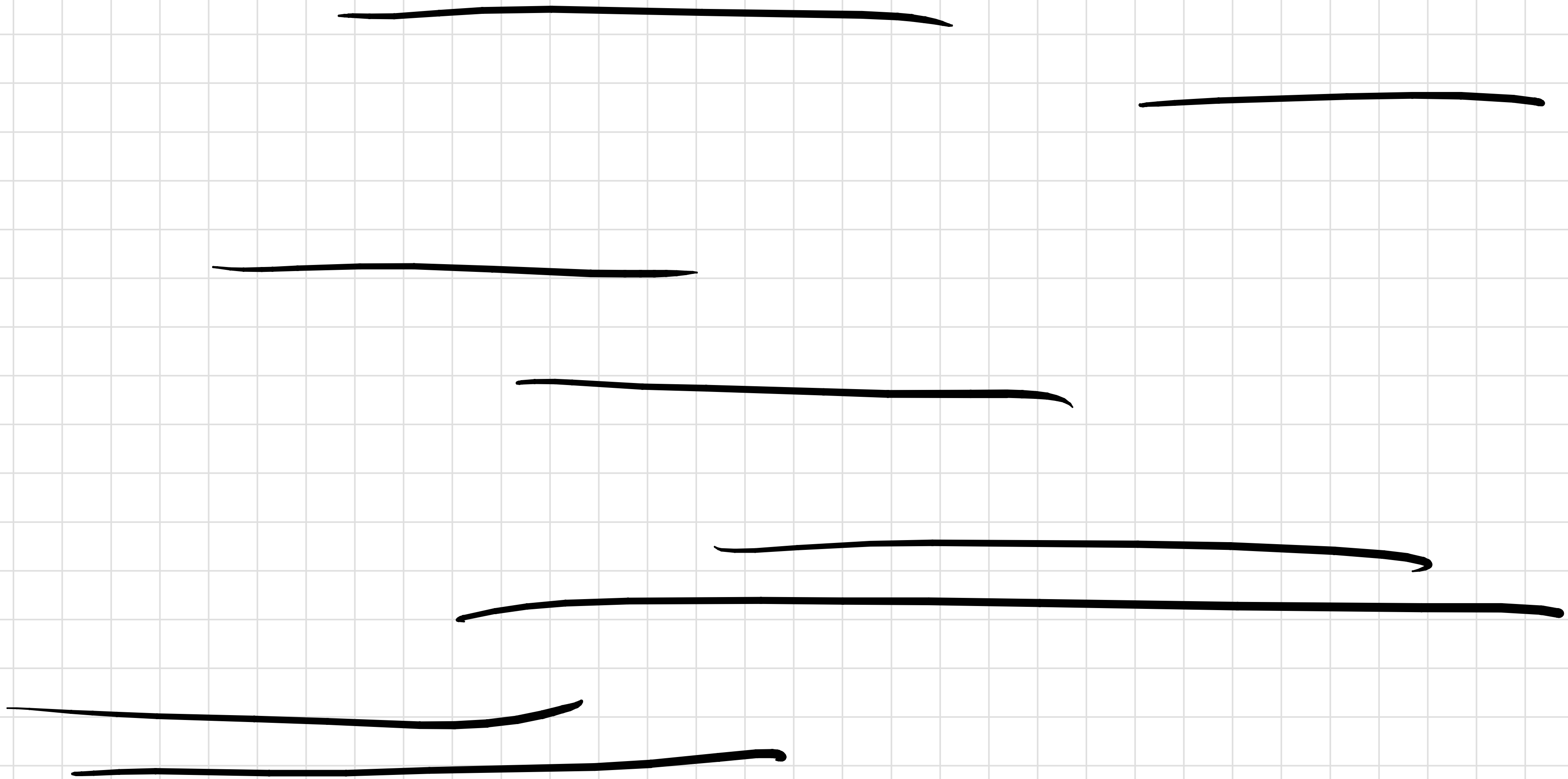
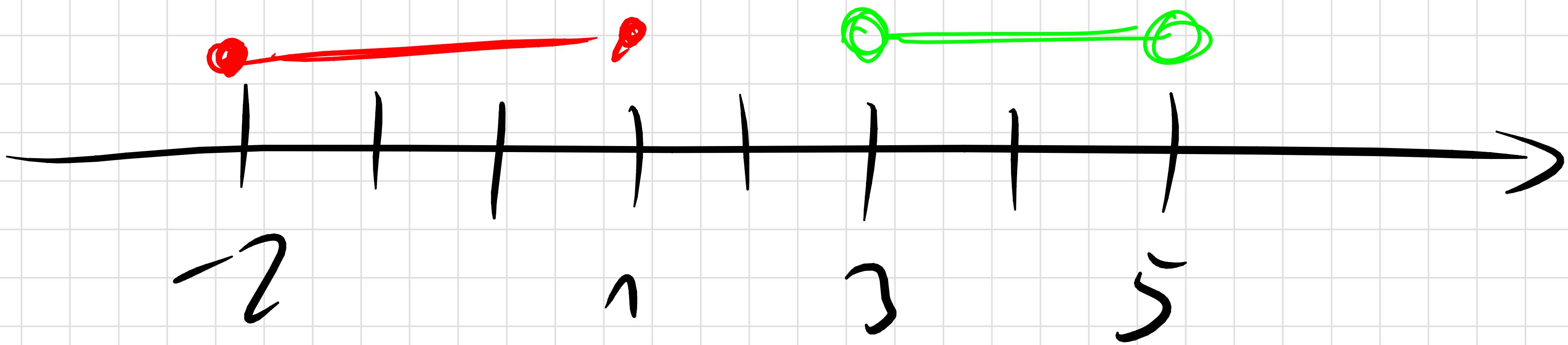
Intervalle Redux

$$(-1; 3] \cap [2; 5) = [2; 3]$$





$[-2; 1] \cup (3; 5)$ = kein Intervall




disjunkt

$$[4;4] = \{4\}$$

$$(4;4) = \emptyset$$

$$\cancel{(4;4]}$$

$$\left[\frac{a-b}{2}, \frac{a+b}{2}\right]$$

$$\cancel{[5, 3]}$$

Quadratische Gleichung

$$3x^2 + 7x - 42 = 0$$

Normalform:

$$x^2 + \underbrace{\boxed{\frac{7}{3}}}_{p}x \underbrace{\boxed{-14}}_{q} = 0$$

$$\left(x + \frac{1}{2}\cdot\frac{7}{3}\right)^2 - \left(\frac{1}{2}\cdot\frac{7}{3}\right)^2 - 14$$

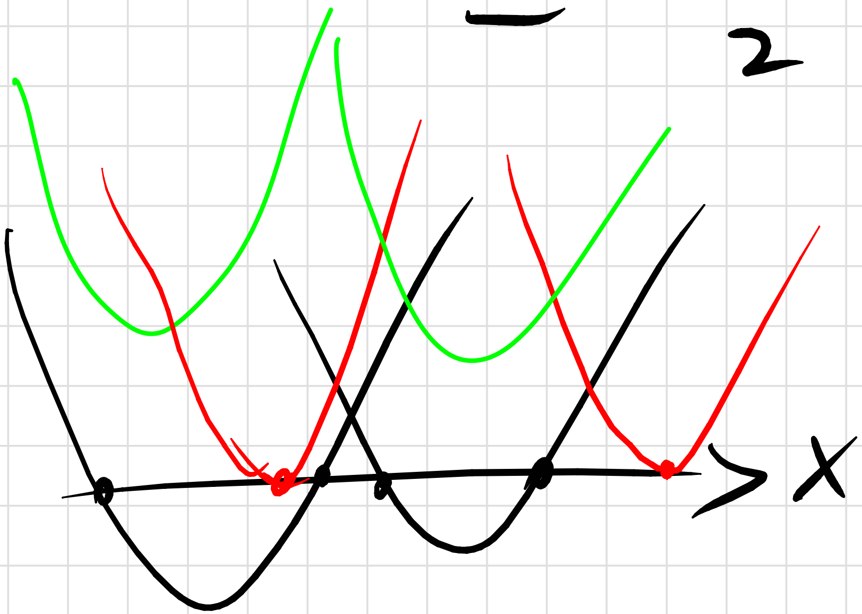
$$x^2 + 2x \cdots + \cdots^2$$

$$\Leftrightarrow \left(x + \frac{1}{2}\cdot\frac{7}{3}\right)^2 = \left(\frac{1}{2}\cdot\frac{7}{3}\right)^2 + 14$$

$$\Leftrightarrow x + \frac{1}{2}\cdot\frac{7}{3} = \pm\sqrt{\left(\frac{1}{2}\cdot\frac{7}{3}\right)^2 + 14}$$

$$\Leftrightarrow \quad x = -\frac{1}{2}\cdot\frac{7}{3} \pm \sqrt{\left(\frac{1}{2}\cdot\frac{7}{3}\right)^2 + 14}$$

$$= -\frac{1}{2}p \pm \sqrt{\underbrace{\left(\frac{1}{2}p\right)^2 - q}}$$



> 0 : 2 Lösungen

$= 0$: 1 Lösung

< 0 : 0 Lösungen

Polynom:

grad

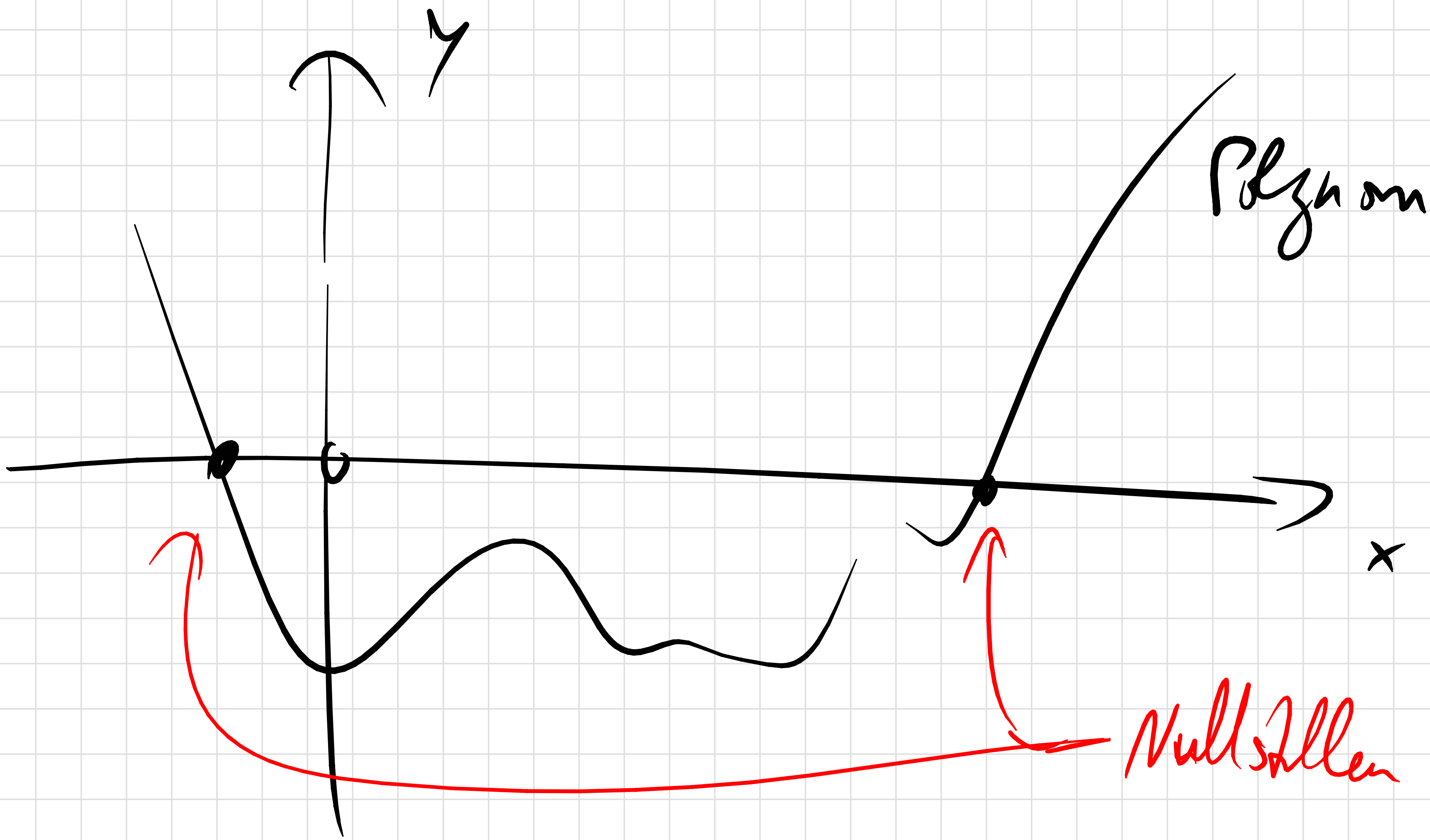
$$\cancel{0 \cdot x^{43}} + 7 \cdot x^{42} + \sqrt{3} \cdot x^{21} - \pi x^{5} + 3x^{1} + 7$$

Koeffizienten

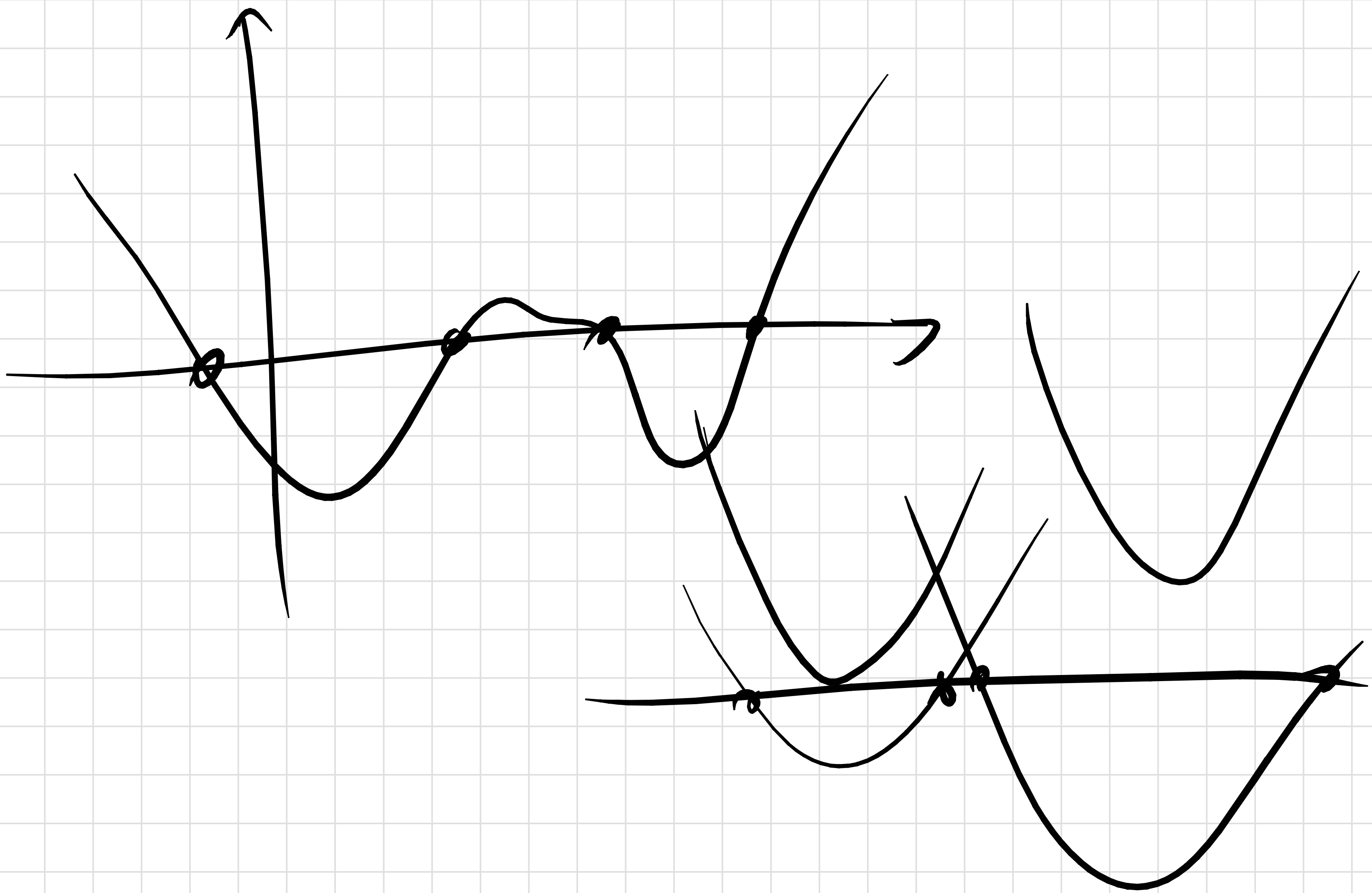
$$\sin(x) \approx x - \frac{x^3}{6} + \frac{x^5}{120} - \ldots \qquad x^{13}$$

$$\text{Polynom}(x_0) = 0$$

Nullstelle



y
Polynom
x
Nullstellen



Polynomdivision

zur Erinnerung:

$$
\begin{array}{rl}
234 : 5 & = 46 \text{ Rest } 4 \\
-20 & \\
\hline
34 & \\
-30 & \\
\hline
4 &
\end{array}
$$

d.h. $\dfrac{234}{5} = 46\,\dfrac{4}{5}$

$234 = 5 \cdot 46 + 4$

$$(x^4 - 2x^3 - x^2 + 3x - 2) : (x+3) = x^3 - 5x^2 + 14x - 39 \quad \text{Rest } 115$$

$$-(x^4 + 3x^3)$$

$$-5x^3 - x^2$$

$$-(-5x^3 - 15x^2)$$

$$14x^2 + 3x$$

$$-(14x^2 + 42x)$$

$$-39x - 2$$

$$-(-39x - 117)$$

$$115$$

Also:

$$x^4 - 2x^3 - x^2 + 3x - 2$$

$$= (x+3)(x^3 - 5x^2 + 14x - 39) + 115$$

Vgl.: $234 = 5 \cdot 46 + 4$

$42 = 6 \cdot 7 + 0$

Teiler

Angenommen, x_0 ist Nullstelle des Polynoms

d.h. $\text{Polynom}(x_0) = 0$.

Dann:

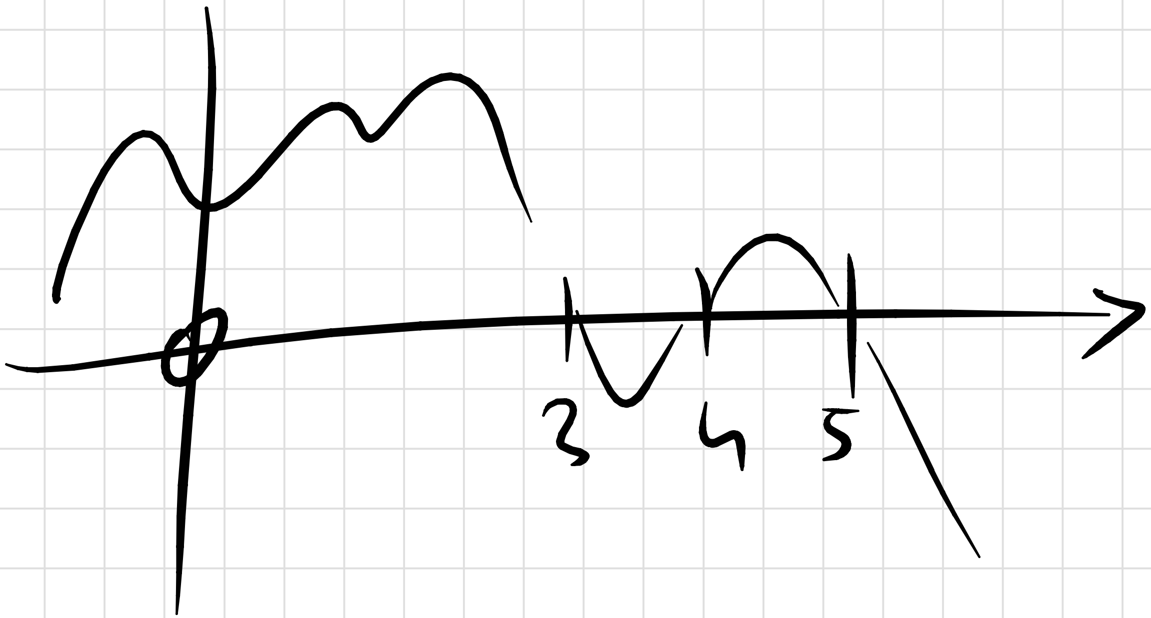
$$\underbrace{\text{Polynom}(x)}_{\text{Grad } n} = \underbrace{(x - x_0)}_{\text{Linearfaktor}} \cdot \overbrace{\text{anderesPolynom}(x)}^{\text{Grad } n-1} + \underbrace{\text{Rest}}_{\text{reine Zahl}}$$

$$\Rightarrow \underbrace{\text{Polynom}(x_0)}_{0} = (x_0 - x_0) \cdot \text{anderesPoly}(x_0) + \text{Rest}$$

$$= 0 \cdot ? + \text{Rest}$$

$$\Rightarrow \text{Rest} = 0$$

Angenommen, $x=3$, $x=4$, $x=5$ sind Nullstellen meines Polynoms.



Muss Nullstellen 4 und 5 haben!

$$\text{Polynom}(x) = (x-3)\cdot \underline{\text{einfacheresPolynom}(x)}$$

$(x-4) \cdot$ noch einfacheres P(x)

$(x-5) \cdot$ nochnoch einfacheres P(x)

$= (x-3)(x-4)(x-5)$

$\cdot$ nochnoch einfacheres Polynom (x)

Allgemein:

$$\text{Polynom}(x) = (x - 1.\,\text{Nullstelle})^{\text{Vielfachheit}} \cdot (x - 2.\,\text{Nullstelle})^{\dots} \cdot \ldots \cdot \text{Polynom ohne Nullstellen}$$

z.B.

$$(x-3)^2 \cdot (x-4)^5 \cdot (x-5)^3 \cdot (x^2+1)$$

$$42 = 2 \cdot 3 \cdot 7$$

Kleine lineare Gleichungssysteme

$$\begin{cases} 2x + 3y = 7 & \mathrm{I} \\ 5x + 4y = 3 & \mathrm{II} \end{cases}$$

Löse I nach y auf: $y = \frac{7-2x}{3}$.

Setze in II ein:

$$5x + \underbrace{4\,\frac{7-2x}{3}}_{\frac{28}{3} - \frac{8}{3}x} = 3$$

$$2\frac{1}{3}x + \frac{28}{3}$$

$$\Leftrightarrow \quad 2\frac{1}{3}x = 3 - \frac{28}{3}$$

$$\Leftrightarrow \quad x = \frac{3 - \frac{28}{3}}{2\frac{1}{3}} = \ldots$$

$$\begin{cases} 8x + 12y = 28 & 4\,\mathrm{I} \\ 15x + 12y = 9 & 3\,\mathrm{II} \end{cases}$$

$$-7x = 19 \qquad 4\,\mathrm{I} - 3\,\mathrm{II}$$

$$\Rightarrow x = -19/7$$

$$2x + 3y = 7 \qquad \mathrm{I}$$

$$\Rightarrow y = \frac{7-2x}{3} = \frac{7 - 2\cdot\left(-\frac{19}{7}\right)}{3} = \;...$$

$$2x + 3y = 7$$
$$\wedge\ 5x + 4y = 3$$

$$\Longleftrightarrow \quad y = \frac{7 - 2x}{3}$$
$$\wedge\ 5x + 4y = 3$$

$$\Longleftrightarrow \quad \ldots$$

$$\begin{cases} x + y = 1 \\ x + 2y = 3 \end{cases}$$

$$\Longleftrightarrow \begin{cases} y = 1 - x \\ x + 2(1 - x) = 3 \end{cases}$$

$$\Longleftrightarrow \begin{cases} y = 1 - x \\ -x = 1 \end{cases} \Longleftrightarrow \begin{cases} y = 2 \\ x = -1 \end{cases}$$

$\mathbb{L} = \{ (-1|2) \}$

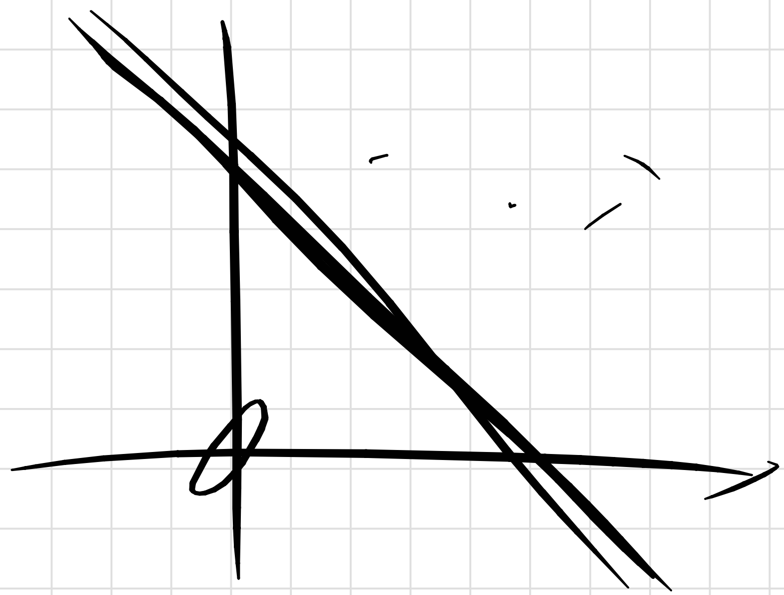
$$\begin{cases} x + y = 2 \\ 2x + 2y = 5 \end{cases}$$

$2\mathrm{I} - \mathrm{II}$:

$0 = -1$

$\mathbb{L} = \emptyset$

$$\begin{cases} x+y=2 \\ 2x+2y=4 \end{cases}$$



$$\mathbb{L} = \left\{ (1|1),\ (2|0),\ (0|2),\ (4|-2),\ \left(\tfrac{1}{2}, \tfrac{3}{2}\right), \ldots \right\}$$

$$= \left\{ (x|y) \in \mathbb{R}^2 : x+y=2 \right\}$$