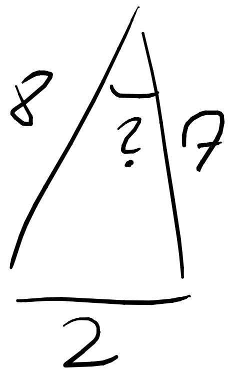
Mathematik I
2011-09-19

1) $10^{-\sqrt{x+3}} = \frac{1}{100} = 10^{-2}$

$\Longleftrightarrow \sqrt{x+3} = 2 \Longleftrightarrow x+3 = 4$

$\Longleftrightarrow x = 1$

2)



Winkel ist eindeutig!

Cosinussatz:

$$\text{Winkel} = \arccos\Big(\underbrace{\frac{8^2+7^2-2^2}{2\cdot 8\cdot 7}}_{\frac{109}{112}}\Big)$$

$((\approx 13^\circ))$

3) $$\frac{x+1}{x^3-x^2} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x-1}$$

$x^2(x-1)$

$$= \frac{A(x-1)+Bx(x-1)+Cx^2}{x^2(x-1)}$$

$\Longrightarrow C=2, A=-1, B=-2$ z.B. $x=2$ einsetzen

$x=1$ einsetzen $x=0$ einsetzen

4) $\dfrac{d \sin\left(\frac{e^x}{\sqrt{x}}\right)}{dx} = \cos\left(\cdot/\cdot\right) \dfrac{e^x \cdot \sqrt{x} - e^x \cdot \frac{1}{2\sqrt{x}}}{x}$

$$\left(\left(= \cos\left(\frac{e^x}{\sqrt{x}}\right) e^x \; \frac{x - \frac{1}{2}}{x^{3/2}} \right)\right)$$

5) $\displaystyle\int_1^3 \ln(x) \cdot \frac{1}{x^2} \, dx$

$\ln(x) \downarrow 1/x \qquad \frac{1}{x^2} \uparrow -x^{-1}$

$$= \left[\ln(x)\left(-\frac{1}{x}\right)\right]_1^3 - \int_1^3 \frac{1}{x} \cdot \left(-\frac{1}{x}\right) dx$$

$$= -\frac{\ln(3)}{3} + 0 + \left[-x^{-1}\right]_1^3$$

$$= -\frac{\ln(3)}{3} - \frac{1}{3} + 1 = \frac{2}{3} - \frac{\ln(3)}{3}$$

6) Variant om X

$$= E[X^2] - (\underbrace{E[X]}_{0})^2$$

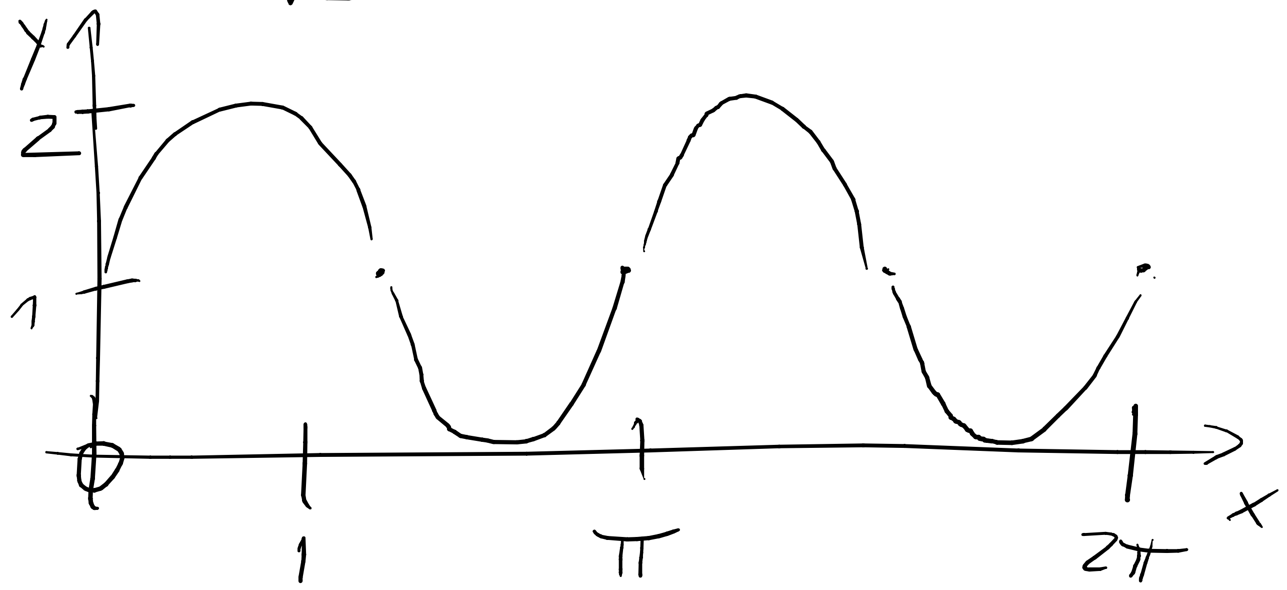
$$= \int_{-1}^{1} x^2 \frac{3}{4}(1-x^2)\,dx$$

$$= \frac{3}{4}\int_{-1}^{1}(x^2-x^4)\,dx = \frac{3}{4}\left[\frac{x^3}{3}-\frac{x^5}{5}\right]_{-1}^{1}$$

$$= \frac{3}{4}\left(\frac{1}{3}-\frac{1}{5}--\frac{1}{3}---\frac{1}{5}\right) = \frac{1}{5}$$

$$\Rightarrow \sigma = \frac{1}{\sqrt{5}}$$

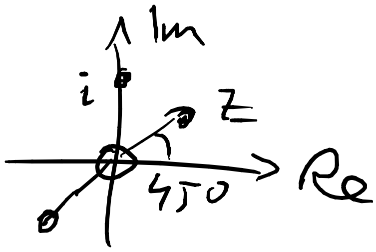
7)



8) $z^4 = iz^2$

$\iff z^2(i - z^2) = 0$

$\iff z = 0 \vee z^2 = i$

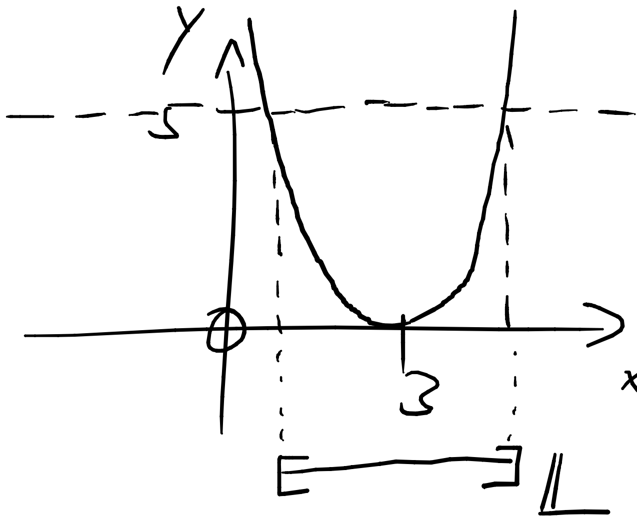


$\iff z = 0 \vee z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

$\vee\ z = -\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$

9) $(x-3)^2 \leq 5$

z.B. so:



x am Rand von $\mathbb{L}$

$\iff (x-3)^2 = 5 \iff x - 3 = \pm\sqrt{5}$

$\iff x = 3 \pm \sqrt{5}$

Also $\mathbb{L} = [3-\sqrt{5}, 3+\sqrt{5}]$

10) Zahl der Möglichkeiten =

wähle die eine doppelte.

Platziere die zweimal.

$$5 \cdot \binom{6}{2} \cdot 4!$$

Platziere die übrigen vier.

$$\left(\left(= 5 \cdot \frac{6 \cdot 5}{\cancel{2} \cdot 1} \cdot 4 \cdot 3 \cdot \cancel{2} = 1800 \right)\right)$$

11) $$\frac{5n^2 + \sin(n^3)}{\cos(n^5) + 3n^2} = \frac{5 + \frac{\sin(n^3)}{n^2}}{\frac{\cos(n^5)}{n^2} + 3} \longrightarrow \frac{5}{3}$$

$\frac{\sin(n^3)}{n^2} \to 0$

$\frac{\cos(n^5)}{n^2} \to 0$

12) Bogenlänge $= \int_0^1 \sqrt{1 + f'(x)^2}\, dx$

$$= \int_0^1 \sqrt{1 + \left(\cancel{2} \cdot \frac{3}{\cancel{2}} (x+1)^{1/2}\right)^2}\, dx$$

$$= \int_0^1 \sqrt{1 + 9(x+1)}\, dx = \int_0^1 \sqrt{9x + 10}\, dx$$

$$= \int_{10}^{19} \sqrt{u}\,\frac{du}{9} = \frac{1}{9}\left[\frac{2}{3}x^{3/2}\right]_{10}^{19}$$

$$u = 9x + 10$$

$$du = 9\,dx$$

$$= \frac{2}{27}\left(19^{3/2} - 10^{3/2}\right)$$

$$\left(\left(\approx 3{,}8\right)\right)$$