

$$1) \begin{pmatrix} y \\ y' \end{pmatrix}' = \begin{pmatrix} y' \\ y'' \end{pmatrix} = \begin{pmatrix} y' \\ -y \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}$$

Also write:

$$\frac{d}{dx} \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_A \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}$$

$$2) \exp\left(x \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right) =$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \frac{x^2}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^2 + \frac{x^3}{6} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^3 + \frac{x^4}{24} (\dots)^4 + \dots$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

AG hier gehts
von vorne los!

$$= \begin{pmatrix} 1 - \frac{x^2}{2} + \frac{x^4}{24} - + \dots & x - \frac{x^3}{6} + - \dots \\ -x + \frac{x^3}{6} - + \dots & 1 - \frac{x^2}{2} + \frac{x^4}{24} - + \dots \end{pmatrix}$$

$$= \begin{pmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{pmatrix} \quad \left(\begin{array}{l} \text{Drehung} \\ \text{um } -x! \end{array} \right)$$

3)

$$f(x) = x^{-1} \xrightarrow{\text{an } x_0 = 3} 1/3$$

$$f'(x) = -x^{-2} \xrightarrow{} -1/9$$

$$f''(x) = 2x^{-3} \xrightarrow{} 2/27$$

Also Schmiegeparabel an $x_0 = 3$:

$$y = \frac{1}{3} - \frac{1}{9}(x-3) + \frac{1}{27} \frac{(x-3)^2}{2}$$