1) Ansatz: $y = \underset{\substack{\uparrow \\ 1 \text{ wegen Anfangsbedingung}}}{a_0} + a_1(x-3) + a_2(x-3)^2 + \ldots$

Einsetzen:

$$a_1 + 2a_2(x-3) + \ldots$$

$$\overset{!}{=} \underset{\substack{\uparrow \\ x-3+3}}{x} \cdot \left(1 + a_1(x-3) + a_2(x-3)^2 + \ldots\right)$$

$$= (x-3) + a_1(x-3)^2 + a_2(x-3)^3 + \ldots$$
$$+ 3 + 3a_1(x-3) + 3a_2(x-3)^2$$

$$\Rightarrow a_1 = 3 \wedge \underbrace{2a_2 = 1 + 3a_1}_{a_2 = 5}$$

2) Trennung der Variablen:

$$\underbrace{\int_1^{y_1} \frac{1}{y}\,dy}_{\ln|y_1| - 0} = \underbrace{\int_3^{x_1} x\,dx}_{\frac{x_1^2}{2} - \frac{9}{2}}$$

$$\Rightarrow \quad y_1 = \pm\, e^{x_1^2/2 - 9/2}$$

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3) $f(x) = e^{\frac{x^2}{2} - \frac{9}{2}}$ entwickeln an $x_0 = 3$:

$$f'(x) = \frac{\cancel{2}x}{\cancel{2}} e^{\frac{x^2}{2} - \frac{9}{2}}, \quad f''(x) = e^{\frac{x^2}{2} - \frac{9}{2}} \cdot (1 + x^2)$$

Also $f(3) = e^0 = 1$, $f'(3) = 3e^0 = 3$, $f''(3) = e^0 \cdot 10 = 10$. Macht:

$$f(x) = 1 + 3(x-3) + \cancel{10}^{\,5} \frac{(x-3)^2}{\cancel{2}} + \dots$$