

1. links: $-y(0) + sY(s) + 3Y(s)$

rechts: $\frac{2}{s^2+4}$

2. $Y(s) = \left(\frac{2}{s^2+4} + y(0) \right) / (s+3)$

$$= \frac{2 + y(0)(s^2+4)}{(s^2+4)(s+3)}$$

$$\stackrel{!}{=} \frac{A + Bs}{s^2+4} + \frac{C}{s+3} \quad \begin{array}{l} \text{Partial-} \\ \text{bruch-} \\ \text{zerlegung} \end{array}$$
$$= \frac{As + 3A + Bs^2 + 3Bs + Cs^2 + 4C}{(s^2+4)(s+3)}$$

$$\left\{ \begin{array}{l} 2 + 4y(0) = 3A + 4C \quad ((s^0)) \\ 0 = A + 3B \quad ((s^1)) \\ y(0) = B + C \quad ((s^2)) \end{array} \right.$$

$$\Leftrightarrow \begin{pmatrix} 3 & 0 & 4 \\ 1 & 3 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 2 + 4y(0) \\ 0 \\ y(0) \end{pmatrix}$$

$$\Leftrightarrow \left\{ \begin{array}{l} A = \frac{6 + 12y(0) - 12y(0)}{9 + 4} = \frac{6}{13} \\ B = \frac{4y(0) - 2 - 4y(0)}{13} = -\frac{2}{13} \end{array} \right.$$

$$C = \frac{9y(0) + 2 + 4y(0)}{13} = y(0) + \frac{2}{13}$$

$$3. \quad \bar{Y}(s) = \frac{\frac{6}{13} - \frac{2}{13}s}{s^2 + 4} + \frac{y(0) + \frac{2}{13}}{s + 3}$$



$$y(t) = \frac{3}{13} \sin(2t) - \frac{2}{13} \cos(2t) + \left(y(0) + \frac{2}{13}\right) e^{-3t}$$

4. allg. Lsg. der homogenen Form:

$$y(t) = D e^{-3t}$$

eine spez. Lsg. der inhomogenen Form:

$$\text{Ansatz: } y(t) = E \sin(2t) + F \cos(2t)$$

$$\Rightarrow y(t) = 2E \cos(2t) - 2F \sin(2t)$$

$$\text{Einsetzen: } 2E \cos(2t) - 2F \sin(2t) + 3E \sin(2t) + 3F \cos(2t) \stackrel{!}{=} \sin(2t)$$

$$\Leftrightarrow \begin{cases} 2E + 3F = 0 \\ 3E - 2F = 1 \end{cases} \Leftrightarrow \begin{cases} E = \frac{-3}{-13} = \frac{3}{13} \\ F = \frac{2}{-13} = -\frac{2}{13} \end{cases}$$



Allg. Lsg der inh. Form:

$$y(t) = D e^{-3t} + \frac{3}{13} \sin(2t) - \frac{2}{13} \cos(2t)$$

$$y(0) = D - \frac{2}{13} \Rightarrow D = y(0) + \frac{2}{13}$$

Damit dasselbe Resultat!