Praktikum 8

1) $f(x) = \ln(x)$, $f'(x) = \frac{1}{x}$, $f''(x) = -\frac{1}{x^2}$

Also:

$$\ln(x) \approx \ln(5) + \frac{1}{5}(x-5) - \frac{1}{25}\frac{(x-5)^2}{2}$$

$\underbrace{\qquad\qquad\qquad}_{\text{Tangentengrade}}$

$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{quadr. Schmiegeparabel}}$

2) $f(x) = 10^x = e^{x \ln(10)}$

$f'(x) = \ln(10) e^{x \ln(10)} = \ln(10) \, 10^x$
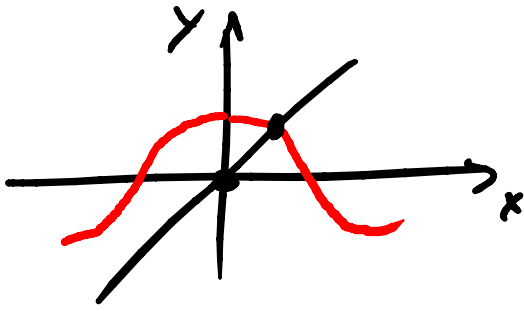
$f''(x) = \left(\ln(10)\right)^2 10^x$

Also: $10^x \approx 10^5 + \ln(10) \, 10^5 (x-5)$

$$+ \left(\ln(10)\right)^2 10^5 \frac{(x-5)^2}{2}$$

$$= 10^5 \left(1 + \ln(10)(x-5) \right.$$
$$\left. + \left(\ln(10)\right)^2 \frac{(x-5)^2}{2} \right)$$

Tangentengrade   alles: quadr. Schmiegeparabel

3)

Näher cos an $x_0 = 0$:

$$\cos(0) = 1$$
$$\cos'(0) = 0$$
$$\cos''(0) = -1$$

$$\Rightarrow \cos(x) \approx 1 - \frac{x^2}{2}$$

Näherungsweise Gleichung:

$$1 - \frac{x^2}{2} = x \iff x^2 + 2x - 2 = 0$$

$$\iff x = -1 \pm \sqrt{1+2} = -1 \pm \sqrt{3}$$

Gemäß der Skizze kommt nur

$x = -1 \,\textcircled{+}\, \sqrt{3}$ in Frage.

$$\left(\!\left(\quad \rightarrow \approx 0{,}732 \quad \right)\!\right)$$
$$\text{Exakt: } 0{,}739\ldots$$