Mathematik 1
2013-01-29

1. $\log_3((1+10^x)^2) = 6$

$\Leftrightarrow (1+10^x)^2 = 3^6 \Leftrightarrow 1+10^x = 3^3 = 27$

((kein Problem mit ±, wie $1+10^x \geq 0$))

$\Leftrightarrow x = \lg(26)$

2. $|x^2-1| \leq 3$

$\Leftrightarrow x^2-1 \geq 0 \wedge |x^2-1| \leq 3$
$\vee x^2-1 < 0 \wedge |x^2-1| \leq 3$

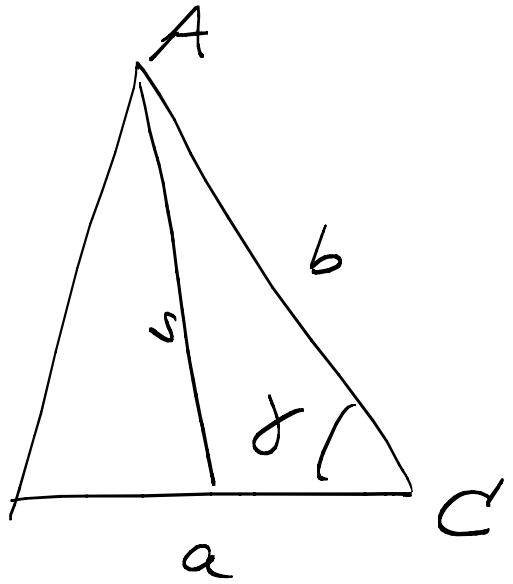
$\Leftrightarrow x^2 \geq 1 \wedge x^2-1 \leq 3$
$\vee x^2 < 1 \wedge -x^2+1 \leq 3$

$\Leftrightarrow x^2 \geq 1 \wedge x^2 \leq 4$
$\vee x^2 < 1 \wedge x^2 \geq -2$

$\Leftrightarrow x^2 \in [1;4] \vee x^2 \in [-2,1)$ $=\mathbb{L}$

$\Leftrightarrow x^2 \in [-2;4] \Leftrightarrow x \in [-2;2]$

3.



Cosinussatz:

$$s^2 = \left(\frac{a}{2}\right)^2 + b^2 - 2\frac{a}{2} b \cos\gamma$$

$$= 2^2 + 5^2 - 20\cos 40°$$

$$\Rightarrow s = \sqrt{29 - 20\cos 40°}$$

4. $$x \mapsto \frac{e^{3x} \cdot 3\sqrt{x^2+1} - (e^{3x}+4)\frac{1}{2\sqrt{x^2+1}} \cdot 2x}{x^2+1}$$

5. $$\left|\frac{1}{5+i}\right| = \frac{1}{|5+i|} = \frac{1}{\sqrt{25+1}} = \frac{1}{\sqrt{26}}$$

6. $$P(\{X \leq \tfrac{3}{2}\}) = \int_1^{3/2} C(x-2)^2 dx$$

N.R.: $C = ?$

$$1 \stackrel{!}{=} \int_1^3 C(x-2)^2 dx = C\left[\tfrac{1}{3}(x-2)^3\right]_1^3$$

$$= \frac{C}{3}(1 - -1) = \frac{2C}{3} \Rightarrow C = \frac{3}{2}$$

$$= \frac{3}{2}\int_1^{3/2} (x-2)^2 dx = \frac{3}{2}\left[\frac{1}{3}(x-2)^3\right]_1^{3/2}$$

$$= \frac{1}{2}\left(\left(-\frac{1}{2}\right)^3 - (-1)^3\right) = \frac{1}{2}\left(-\frac{1}{8}+1\right) = \frac{7}{16}$$

7. $\underbrace{(z^2+1)(z^2-1)}_{z^4-1} = 15$ ((3. Binomi))

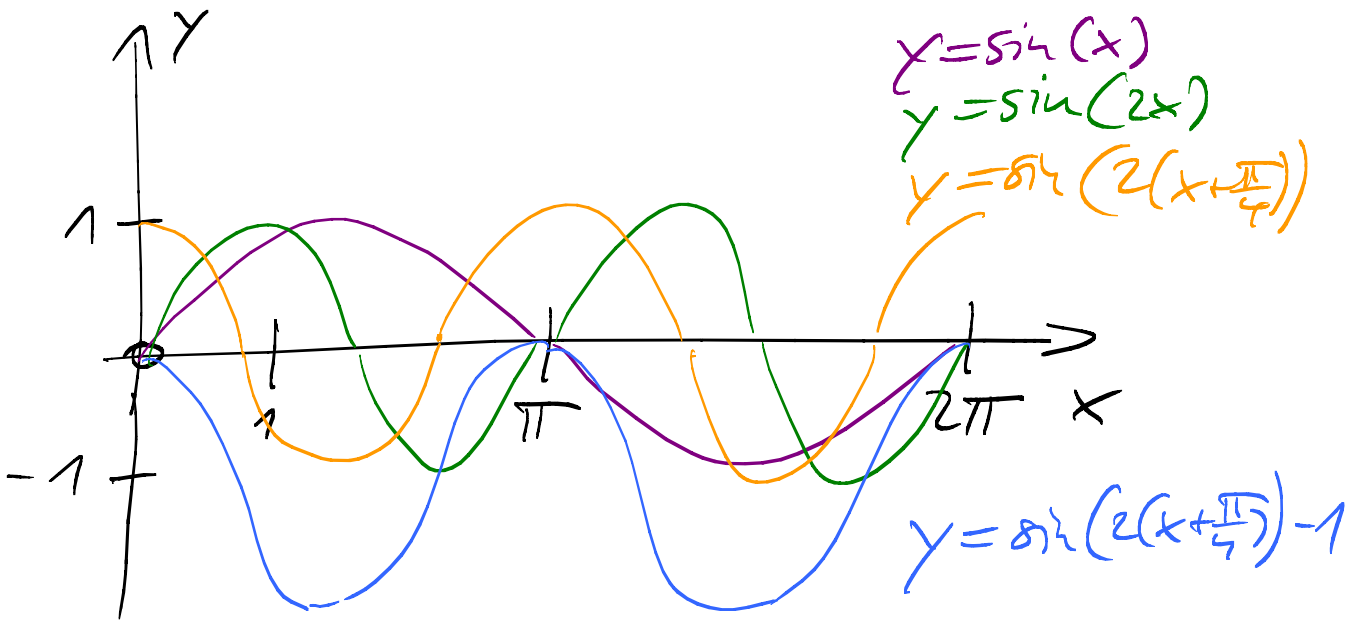
$\Leftrightarrow z^4 = 16 \Leftrightarrow z = 2+0i$

$\vee\ z = -2+0i$

$\vee\ z = 0+2i$

$\vee\ z = 0-2i$

8.



9. $\int_2^3 \left(\frac{x-1}{x+1}\right) dx$

$N.R.:$
$(x-1):(x+1) = 1$ Rest
$-(x+1)$
-2

$\Vert$

$1 - \frac{2}{x+1}$

$= \int_2^3 1\,dx - 2\int_2^3 \frac{dx}{x+1} = 1 - 2\left[\ln|x+1|\right]_2^3$ ↓

$$= 1 - 2\underbrace{\left(\ln(4) - \ln(3)\right)}_{\ln\left(\frac{4}{3}\right)} = 1 - 2\ln\left(\frac{4}{3}\right)$$

$$\left(\left(= 1 - \ln\left(\frac{16}{9}\right)\right)\right)$$

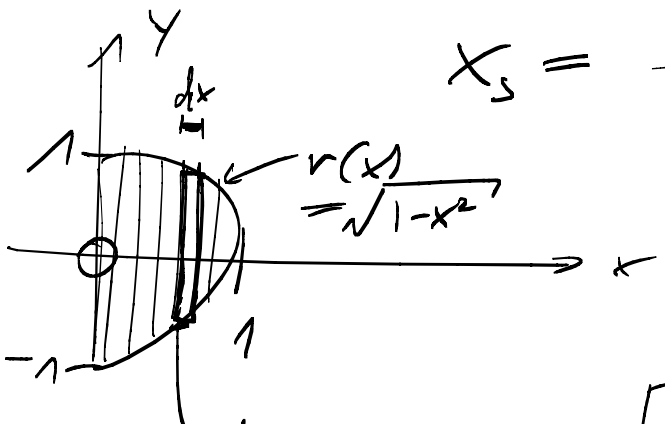
10. $\dfrac{n^2 \sin(n) + 13n}{1+n^2} = \dfrac{\sin(n) + \frac{13}{n}}{\frac{1}{n^2} + 1}$

$\sin(n)$ oszilliert; $\frac{13}{n} \to 0$; $\frac{1}{n^2} \to 0$; Nenner $\to 1$

Konvergiert nicht!

11. $P = \binom{100}{42} \frac{1}{26^{42}} \left(1 - \frac{1}{26}\right)^{58}$

12.



Scheibe mit Volumen $\pi r(x)^2 dx$

$$x_s = \frac{\int_0^1 x \pi r^2(x) dx}{\int_0^1 \pi r^2(x) dx} = \frac{\int_0^1 x(1-x^2) dx}{\int_0^1 (1-x^2) dx}$$

$$= \frac{\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1}{\left[x - \frac{x^3}{3}\right]_0^1} = \frac{\frac{1}{2} - \frac{1}{4}}{1 - \frac{1}{3}} = \frac{3}{8}$$