Mathematik II

2013-01-29

1. $$E_2: \overset{A}{\begin{pmatrix}3\\2\\1\end{pmatrix}} + \lambda \underbrace{\overset{B\quad A}{\begin{pmatrix}4-3\\3-2\\1-1\end{pmatrix}}}_{\begin{pmatrix}1\\1\\0\end{pmatrix}} + \mu \underbrace{\begin{pmatrix}1\\1\\0\end{pmatrix} \times \underbrace{\overset{C\quad A}{\begin{pmatrix}5-3\\3-2\\2-1\end{pmatrix}}}_{\begin{matrix}2\\1\\1\end{matrix}}}_{\begin{pmatrix}1\\-1\\-1\end{pmatrix}}$$

2. $$\begin{vmatrix} 0_+ & 1_- & 0_+ & 1 \\ 1 & 0 & 2_- & \cancel{X}-1 \\ 0 & 0 & 1^+ & \cancel{X}0 \\ -1 & 3 & 1 & \cancel{X}0 \end{vmatrix} = 1 \cdot \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & 3 & 0 \end{vmatrix}$$

$$= 0 + 1 + 3 - 0 - 0 - 0 = 4$$

3. Ansatz: $y(x) = A\sin(x) + B\cos(x)$

$$\Longrightarrow A\cos(x) - B\sin(x) + 2A\sin(x) + 2B\cos(x) = \sin(x)$$

$$\Longrightarrow 2A - B = 1 \wedge 2B + A = 0$$

$$\Longrightarrow B = -\frac{1}{5} \wedge A = \frac{2}{5}$$

4. $y' = e^{x+y} = e^x e^y$

$\Rightarrow \dfrac{y'}{e^y} = e^x$

$$\Rightarrow \underbrace{\int_{2}^{y_1} e^{-y}\,dy}_{\left[-e^{-y}\right]_2^{y_1}} = \underbrace{\int_{1}^{x_1} e^{x}\,dx}_{\left[e^{x}\right]_1^{x_1}}$$

$\Rightarrow -e^{-y_1} + e^{-2} = e^{x_1} - e$

$\Rightarrow y_1 = -\ln\left(-e^{x_1} + e + e^{-2}\right)$

5. $$\frac{s^2}{(s^2+1)(s-3)} = \frac{As+B}{s^2+1} + \frac{C}{s-3}$$

$$= \frac{(As+B)(s-3) + C(s^2+1)}{(s^2+1)(s-3)}$$

$\Rightarrow s^2 = (As+B)(s-3) + C(s^2+1) \quad \forall s$

wähle $s=3$: dann klar, dass $C = 9/10$

wähle $s=0$: dann klar, dass $0 = -3B + 9/10 \Rightarrow B = 3/10$

Koeffizient von s^2: $1 = A + C$

also $A = 1/10$

$$\Rightarrow Y = \frac{\frac{1}{10}s + \frac{3}{10}}{s^2+1} + \frac{\frac{9}{10}}{s-3}$$

$$\frac{1}{10}\cos(t) + \frac{3}{10}\sin(t) + \frac{9}{10}e^{3t}$$

6. $\dfrac{\partial f}{\partial x} = \sqrt[3]{y}$

Wert an $(3/8)$: $\sqrt[3]{8} = 2$

$\dfrac{\partial f}{\partial y} = x\,\frac{1}{3}\,y^{-2/3}$

$3 \cdot \frac{1}{3} \cdot \frac{1}{2^2} = \frac{1}{4}$

$$f(3{,}05;\, 8{,}01) \approx 3\sqrt[3]{8} + 2 \cdot 0{,}05 + \frac{1}{4} \cdot 0{,}01$$

$$= 6 + 0{,}1 + 0{,}0025$$

$$= 6{,}1025$$

7. zum Beispiel: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

8. zum Beispiel: $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

9. $f(x) = \sin\left(\frac{1-x}{1+x}\right)$

$$f'(x) = \cos\left(\frac{1-x}{1+x}\right) \cdot \frac{-1\cdot(1+x) - (1-x)\cdot 1}{(1+x)^2}$$

$$= \cos\left(\frac{1-x}{1+x}\right) \cdot \frac{-1 \cancel{-x} - 1 \cancel{+x}}{(1+x)^2}$$

$$= -2 \, \frac{\cos\left(\frac{1-x}{1+x}\right)}{(1+x)^2}$$

$$f''(x) = -2 \, \frac{-\sin\left(\frac{1-x}{1+x}\right) \cdot \frac{-2}{(1+x)^2} \cdot (1+x)^2 - \cos\left(\frac{1-x}{1+x}\right) 2(1+x)}{(1+x)^4}$$

$f(1) = 0$

$f'(1) = -2 \, \frac{1}{2^2} = -\frac{1}{2}$

$f''(1) = -2 \, \frac{-0 - 1\cdot 2 \cdot 2}{2^4} = \frac{1}{2}$

$$f(0{,}99) \approx 0 - \frac{1}{2} \cdot -0{,}01 + \frac{1}{2} \cdot \frac{0{,}01^2}{2}$$

$$\Big(\Big(= 0{,}005 + 0{,}000025$$
$$= 0{,}005025 \Big)\Big)$$

10. $y'' + 2y' + y = 0$

Ansatz: $y = e^{\lambda x} \Rightarrow \lambda^2 + 2\lambda + 1 = 0$

$\Rightarrow \lambda = -1 \pm \sqrt{1-1} = -1$

Also allg. Lösung $y(x) = Ae^{-x} + Bxe^{-x}$

$$y'' + 2y' + y = 13$$

spezielle Lösung: $y(x) = 13$

$\Rightarrow$ allg. Lösung insgesamt:

$$y(x) = \underset{\substack{\downarrow \\ 0}}{Ae^{-x}} + \underset{\substack{\downarrow \\ 0}}{Bxe^{-x}} + 13$$

Alle Lösungen gehen für $x \to \infty$ gegen 13.

11. gerade Funktion

$\Rightarrow$ kein Sinus $\Rightarrow b_5 = 0$

$$a_5 = \frac{\cancel{2}\,1}{\cancel{6}\,3} \int_{-3}^{3} |t| \cos\left(\cancel{2}\pi\, 5 \cdot \frac{t}{\cancel{6}\,3}\right) dt$$

$$= \frac{2}{3} \int_0^3 t \cos\left(\tfrac{5}{3}\pi t\right) dt \quad \left(\text{wg. } \overleftrightarrow{\text{Symmetrie}}\right)$$

$$t \to 1 \qquad \cos\left(\tfrac{5}{3}\pi t\right) \leftarrow \frac{3}{5\pi} \sin\left(\tfrac{5}{3}\pi t\right)$$

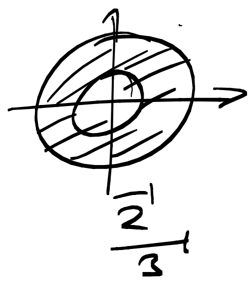
$$= \frac{2}{3}\left(\left[t \frac{3}{5\pi} \sin\left(\tfrac{5}{3}\pi t\right)\right]_0^3 - \int_0^3 \frac{3}{5\pi} \sin\left(\tfrac{5}{3}\pi t\right) dt\right)$$

$$= \frac{2}{3}\left(\underbrace{\dots \sin(5\pi)}_{0} - 0 - \frac{3}{5\pi} \underbrace{\int_0^3 \sin\left(\frac{5}{3}\pi t\right) dt}_{\left[-\frac{3}{5\pi}\cos\left(\frac{5}{3}\pi t\right)\right]_0^3} \right)$$

$$= \frac{2\cdot 9}{3\cdot 25\pi^2}\left(\underbrace{\cos(5\pi)}_{-1} - \cos(0)\right)$$

$$= \frac{-2\cdot \cancel{9}^{\,3}\cdot 2}{\cancel{3}\cdot 25\pi^2} = \frac{-12}{25\pi^2}$$

12. $\iint x^2\,dx\,dy = \int_2^3 r\,dr \int_0^{2\pi} d\varphi\; r^2\cos^2\varphi$



$$= \underbrace{\int_2^3 r^3\,dr}_{\left[\frac{r^4}{4}\right]_2^3}\; \underbrace{\int_0^{2\pi} \cos^2\varphi\,d\varphi}_{\pi}$$

$$= \left(\frac{81}{4} - \frac{16}{4}\right)\cdot\pi = \frac{65}{4}\pi$$