# Mathematik I vom 2014-07-04

## Musterlösungen

1) $\sqrt[3]{5^{x+1}+7} = 2$

$\Leftrightarrow 5^{x+1}+7 = 8$

$\Leftrightarrow 5^{x+1} = 1$

$\Leftrightarrow x+1 = 0$

$\Leftrightarrow x = -1$

2) $z^3 = 2+3i$

$$= \underbrace{\sqrt{2^2+3^2}}_{\sqrt{13}}\, e^{i\cdot \arctan\left(\frac{3}{2}\right)}$$

((Korrekt, weil Realteil $>0$))

Drei Möglichkeiten für $z$:

| Länge | Winkel |
| --- | --- |
| $\sqrt[6]{13}$ | $\frac{1}{3} i \arctan\left(\frac{3}{2}\right)$ |
| " | " $+ \frac{2}{3}\pi$ |
| " | " $+ \frac{4}{3}\pi$ |

3) Wenn $x \to 2$,
dann Zähler $\to 0$
und Nenner $\to 0$

z.B. per L'Hospital:

Zähler' $= 3$

Nenner' $= 2x - 5 \to -1$

Also existiert Grenzwert
und ist $= \frac{3}{-1} = -3$

((Alternative: Nullstellen
des Nenners suchen: $x^2 - 5x + 6 = 0$

$\Leftrightarrow x = \frac{5}{2} \pm \sqrt{\frac{25}{4} - 6}$

$\Leftrightarrow x = 2 \vee x = 3$

$\Leftrightarrow f(x) = \frac{3\cancel{(x-2)}}{\cancel{(x-2)}(x-3)} \to \frac{3}{-1} = -3$))

4) $\frac{d \frac{\sin(3x)}{\sqrt{x}}}{dx} = \frac{\sqrt{x}\cos(3x) \cdot 3 - \frac{1}{2\sqrt{x}}\sin(3x)}{x}$

5) $\int_0^3 x \sqrt{x+7}\, dx$

$x \downarrow 1$ $\qquad$ $\sqrt{x+7} \uparrow \frac{2}{3}(x+7)^{3/2}$

$\downarrow$

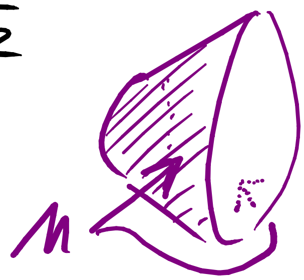
$$= \left[ x \, \frac{2}{3} (x+7)^{3/2} \right]_0^3 - \int_0^3 \frac{2}{3} (x+7)^{3/2} \, dx$$

$$= \cancel{3} \cdot \frac{2}{\cancel{3}} (3+7)^{3/2} - 0$$

$$- \left[ \frac{2}{3} \cdot \frac{2}{5} (x+7)^{5/2} \right]_0^3$$

$$= 2 \cdot 10^{3/2} - \frac{4}{15} \left( 10^{5/2} - 7^{5/2} \right)$$

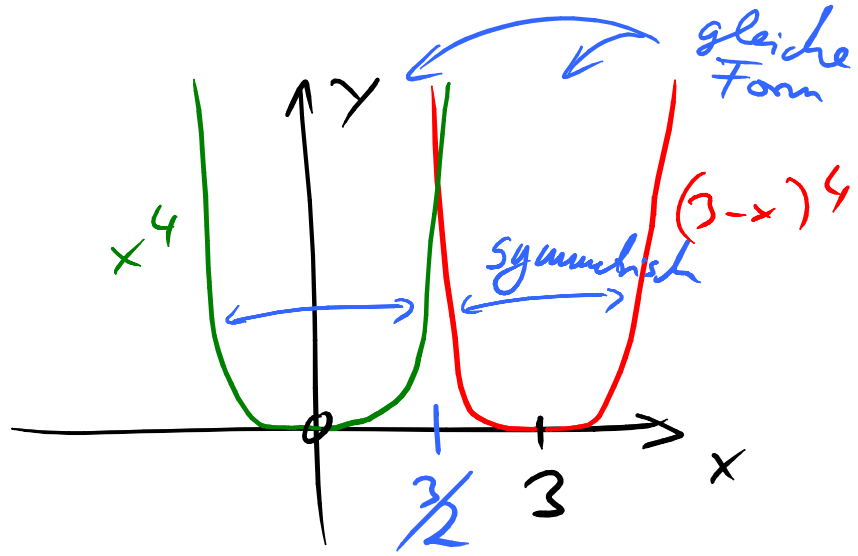
$$6) \quad M = \int_0^5 2\pi (x+3) \underbrace{\sqrt{1 + \left( \frac{d(x+3)}{dx} \right)^2}}_{\sqrt{2}} \, dx$$




$$= 2\sqrt{2}\pi \left[ \frac{x^2}{2} + 3x \right]_0^5$$

$$= 2\sqrt{2}\pi \left( \frac{25}{2} + 15 - 0 \right) = 55\sqrt{2}\pi$$

7)




Also $\mathbb{L} = (-\infty, \tfrac{3}{2})$.

((Alternative: $(3-x)^4 > x^4$

$\iff |3-x| > |x|$

$\iff 3-x \geq 0 \wedge x \geq 0 \wedge 3-x > x$

$\vee\ 3-x < 0 \wedge x \geq 0 \wedge x-3 > x$

$\vee\ 3-x \geq 0 \wedge x < 0 \wedge 3-x > -x$

$\vee\ 3-x < 0 \wedge x < 0 \wedge x-3 > -x$

$\iff x \leq 3 \wedge x \geq 0 \wedge 3 > 2x$

$\vee\ x > 3 \wedge x \geq 0 \wedge -3 > 0$ immer falsch

$\vee\ x \leq 3 \wedge x < 0 \wedge 3 > 0$ immer wahr

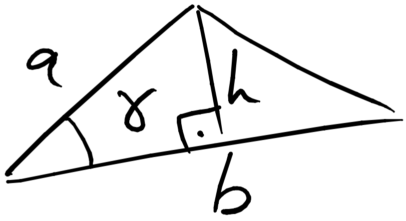
$\vee\ x > 3 \wedge x < 0 \wedge 2x > 3$ immer falsch

$\iff x \geq 0 \wedge x < \frac{3}{2}$

$\vee\ x < 0$

$\iff x < \frac{3}{2}$ ))

8) $P = \dfrac{\binom{10}{6}}{\binom{49}{6}} = \dfrac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5}{49\cdot 48\cdot 47\cdot 46\cdot 45\cdot 44}$

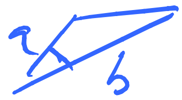
9) 


$h = a \sin \gamma$

$A = \frac{1}{2} a b \sin \gamma$

$$\Rightarrow \sin \gamma = \frac{2 \cdot 2}{3 \cdot 5}$$

$$\Rightarrow \gamma = \arcsin\left(\frac{4}{15}\right)$$




$$\text{oder } \gamma = 180^\circ - \gamma$$


10) Zum Beispiel:

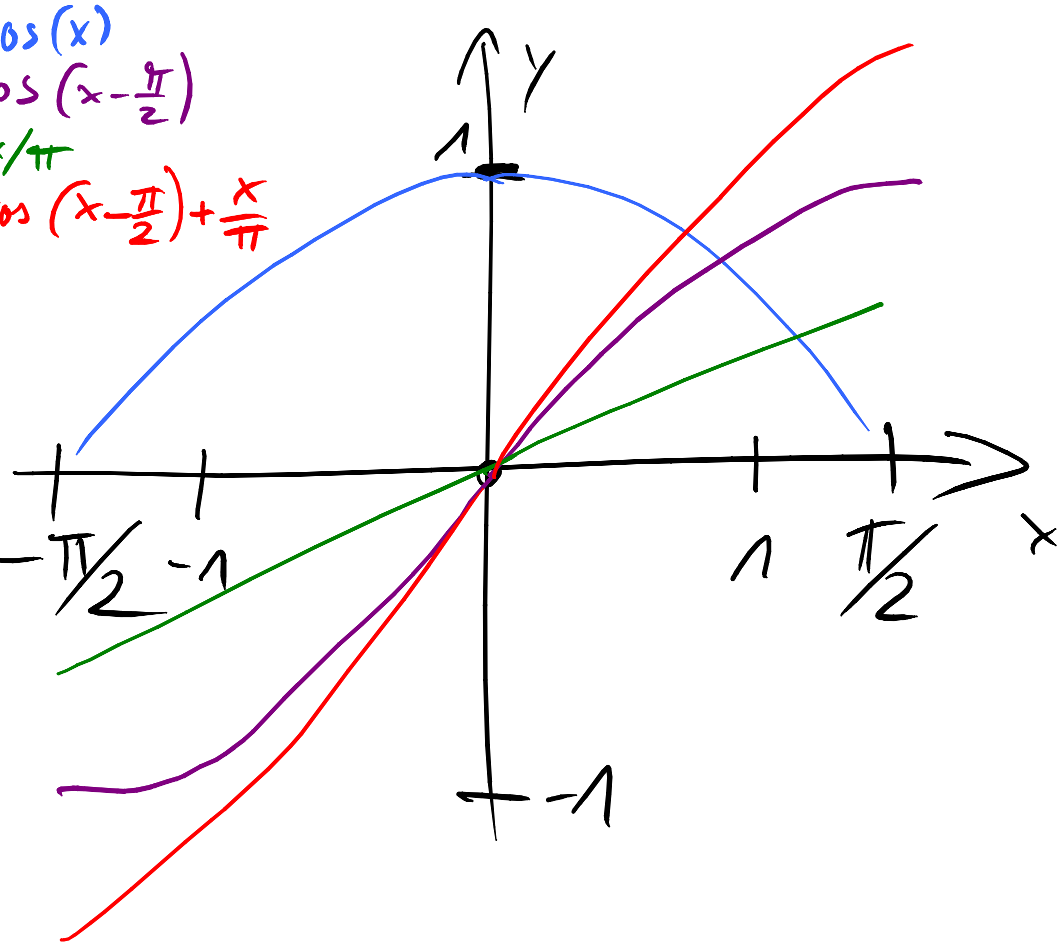
$$f(x) = 2x + \frac{7 \cdot 9}{(x-3)^2}$$

11) $\cos(x)$

$\cos\left(x - \frac{\pi}{2}\right)$

$x/\pi$

$\cos\left(x - \frac{\pi}{2}\right) + \frac{x}{\pi}$



12) $X$ hat

den Wert mit der Wahrscheinlichkeit:

| | |
|---|---|
| $0$ | $1/8$ |
| $1/3$ | $3/8$ |
| $2/3$ | $3/8$ |
| $1$ | $1/8$ |

$$E[X] = 0 \cdot \frac{1}{8} + \frac{1}{3} \cdot \frac{3}{8} + \frac{2}{3} \cdot \frac{3}{8} + 1 \cdot \frac{1}{8}$$

$$= \frac{12}{24} = \frac{1}{2}$$ ((Hätte man auch sofort sagen können.))

$$E[X^2] = 0 \frac{1}{8} + \frac{1}{9} \cdot \frac{3}{8} + \frac{4}{9} \cdot \frac{3}{8} + 1 \cdot \frac{1}{8}$$

$$= \frac{3 + 12 + 9}{8 \cdot 9} = \frac{\cancel{24}^{\cancel{3}\,1}}{\cancel{8}\cancel{9}_{1\;3}} = \frac{1}{3}$$

$$\sigma = \sqrt{\frac{1}{3} - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{4-3}{12}} = \frac{1}{\sqrt{12}}$$

$$\left(\left( = \frac{1}{2\sqrt{3}} \right)\right)$$