Mathematik 2017-07-27

Musterlösungen

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1) Ebene: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

y-Achse: $x = 0 = z$

Schnittmenge: $\begin{cases} 0 = 3 - \lambda + \mu \\ 0 = 1 + 2\lambda + 3\mu \end{cases}$

$$\Longleftrightarrow \begin{cases} \lambda = \dfrac{\begin{vmatrix} -3 & 1 \\ -1 & 3 \end{vmatrix}}{\begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix}} = \dfrac{-8}{-5} = \dfrac{8}{5} \\[2ex] \mu = \dfrac{\begin{vmatrix} -1 & -3 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix}} = \dfrac{7}{-5} = -\dfrac{7}{5} \end{cases}$$

Also $y = 2 + \frac{8}{5} \cdot 3 - \frac{7}{5} \cdot 2 = 4$.

Also Schnittmenge $= \{(0|4|0)\}$.

2) Formel: $\left\{ \begin{array}{cccc} 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 2 \\ 3 & -4 & 3 & 3 \\ 0 & 1 & 3 & 4 \end{array} \right. \quad \begin{array}{l} \\ \cdot(-2) \\ \cdot(-3) \\ \\ \end{array}$

$$\Longleftrightarrow \left\{ \begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & -7 & 3 & 0 \\ 0 & 1 & 3 & 4 \end{array} \right. \quad \begin{array}{l} \\ \\ \cdot(-7) \\ \cdot 1 \end{array}$$

$$\Longleftrightarrow \left\{\begin{matrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 4 & 4 \end{matrix}\right.$$

$$\Longleftrightarrow \left\{\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 4 \end{matrix}\right. \quad \lightning$$

Es gibt keine Lösung!

3) $y' - y \overset{\circledast}{=} 3$

- allg. Lsg. der homogenen Form $y' - y = 0$:
  $y = Ae^x$
- eine spez. Lsg. von $\circledast$:
  $y = -3$
- allg. Lsg. von $\circledast$:
  $y = Ae^x - 3$
- Anfangsbedingung:
  $5 \overset{!}{=} Ae^1 - 3 \Rightarrow A = 8/e$

4) $\frac{y'}{y} = x^3$, also $\underbrace{\int_5^{y_1} \frac{dy}{y}}_{\left[\ln|y|\right]_5^{y_1}} = \underbrace{\int_1^{x_1} x^3 dx}_{\left[\frac{x^4}{4}\right]_1^{x_1}}$

$\Rightarrow |y_1| = 5e^{(x_1^4 - 1)/4}$

Betragsstriche können hier entfallen.

5) $f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$\sqrt{103} \approx \sqrt{100} + \frac{1}{2\sqrt{100}} \cdot 3 = 10{,}15$

6) $V = \int_0^{\pi/4} d\varphi \underbrace{\int_0^3 dr\; r \cdot (10 + r)}_{\left[5r^2 + \frac{r^3}{3}\right]_0^3}$

$= \frac{\pi}{4}\left(45 + \frac{\cancel{3} \cdot 9}{\cancel{3}}\right) = \frac{27}{2}\pi$

7) $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

8) Bild = Ebene $\lambda \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

Rang $= 2 \neq 3$, d.h. LGS nicht für alle rechten Seiten lösbar.

9) 5 ist E.W. $\Longleftrightarrow$

$$0 = \begin{vmatrix} 1-5 & 2 & 0 & 0 \\ 3 & 4-5 & 0 & 0 \\ 0 & 0 & 3-5 & 0 \\ 0 & 5 & 2 & 1-5 \end{vmatrix} = \begin{vmatrix} -4 & 2 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 5 & 2 & -4 \end{vmatrix}$$

$$= -4 \begin{vmatrix} -4 & 2 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & -2 \end{vmatrix} = \underbrace{(-4)\cdot(-2)}_{\neq 0} \cdot \underbrace{\begin{vmatrix} -4 & 2 \\ 3 & -1 \end{vmatrix}}_{\neq 0}$$

Also ist 5 <u>kein</u> E.W. !

10) $y'' - 6y' + 9y \stackrel{!}{=} 0$

Ansatz: $y(x) = e^{\lambda x}$

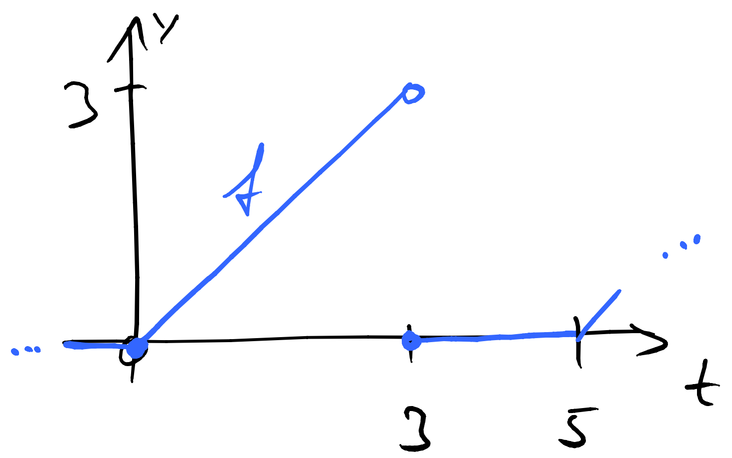
$\to \lambda^2 \cancel{e^{\lambda x}} - 6\lambda \cancel{e^{\lambda x}} + 9 \cancel{e^{\lambda x}} = 0$

$\Longleftrightarrow \lambda = 3 \pm \sqrt{9-9} = 3$

Nur ein λ !

Also allg. Lsg: $y(x) = Ae^{3x} + Bxe^{3x}$

11)



$$a_0 = \frac{2}{5} \int_0^5 f(t)\,dt = \frac{2}{5} \cdot \frac{9}{2} = \frac{9}{5}$$

$\downarrow$

$$b_3 = \frac{2}{5}\int_0^5 \sin\left(2\pi \cdot 3t/5\right) f(t)\,dt$$

$$= \frac{2}{5}\int_0^{3} \sin\left(\frac{6\pi}{5}t\right)\cdot t\,dt$$

$$-\frac{5}{6\pi}\cos\left(\frac{6\pi}{5}t\right) \qquad 1$$

$$= \frac{2}{5}\left(\left[-\frac{5}{6\pi}\cos\left(\frac{6\pi}{5}t\right)\cdot t\right]_0^3 - \int_0^3\left(-\frac{5}{6\pi}\right)\cos\left(\frac{6\pi}{5}t\right)dt\right)$$

$$= \frac{2}{5}\left(-\frac{5}{\cancel{6}_{2}\pi}\cos\left(\frac{18}{5}\pi\right)\cdot\cancel{3} - 0 + \frac{5}{6\pi}\left[\frac{5}{6\pi}\sin\left(\frac{6\pi}{5}t\right)\right]_0^3\right)$$

$$= -\frac{1}{\pi}\cos\left(\frac{\cancel{18}^{-2}}{5}\pi\right) + \frac{\cancel{2}}{\cancel{5}}\cdot\frac{\cancel{5}}{{}_3\cancel{6}\pi}\cdot\left(\frac{5}{6\pi}\sin\left(\frac{\cancel{18}^{-2}}{5}\pi\right) - 0\right)$$

$$\left(\left(= -\frac{1}{\pi}\cos\left(\frac{2}{5}\pi\right) - \frac{5}{18\pi^2}\sin\left(\frac{2}{5}\pi\right)\right)\right)$$

12) $$\frac{1}{(s^2+4)(s+1)} = \frac{As+B}{s^2+4} + \frac{C}{s+1}$$

$$= \frac{(As+B)(s+1) + C(s^2+4)}{(s^2+4)(s+1)}$$

Also $1 = (As+B)(s+1) + C(s^2+4)\quad \forall s.$

$$\Leftrightarrow \begin{cases} 0 = A + C \\ 0 = A + B \\ 1 = B + 4C \end{cases} \Leftrightarrow \begin{cases} A = -B \\ B = C \\ C = 1/5 \end{cases}$$

Damit Originalfunktion:

$$y(t) = -\frac{1}{5}\cos(2t) + \frac{1}{10}\sin(2t) + \frac{1}{5}e^{-t}$$