Mathematik 2

2022-09-30

Musterlösungen

1) z.B. $\begin{pmatrix} 3 & 0 & 6 \\ 2 & 0 & 4 \\ 1 & 0 & 2 \end{pmatrix}$

2)

$$\begin{array}{cccc|l} 1 & 1 & 0 & 1 & \\ 2 & 0 & 1 & 2 & \cdot(-2) \\ 0 & -8 & 5 & 3 & \\ 0 & -2 & 3 & 1 & \end{array} \Longleftrightarrow \begin{array}{cccc|l} 1 & 1 & 0 & 1 & \\ 0 & -2 & 1 & 0 & \\ 0 & -8 & 5 & 3 & \cdot(-4) \\ 0 & -2 & 3 & 1 & \cdot(-1) \end{array}$$

$$\Longleftrightarrow \begin{array}{cccc|l} 1 & 1 & 0 & 1 & \\ 0 & -1 & 1 & 0 & \\ 0 & 0 & 1 & 3 & \\ 0 & 0 & 2 & 1 & \cdot(-2) \end{array} \Longleftrightarrow \begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & -5 \end{array} \quad ↯$$

Es gibt keine Lösung.

3) $y^2 y' + x^3 = 0 \Longleftrightarrow y^2 y' = -x^3$

$\Longleftrightarrow y^2 \, dy = -x^3 \, dx$

Also $\int_7^{y_1} y^2 \, dy = -\int_5^{x_1} x^3 \, dx \Longrightarrow y_1 =$

$$\frac{y_1^3}{3} - \frac{7^3}{3} \qquad -\frac{x_1^4}{4} + \frac{5^4}{4} \qquad \sqrt[3]{7^3 - \frac{3}{4}x_1^4 + \frac{3}{4}5^4}.$$

4) $\dot{x} + 7x = \sin(3t)$

Ansatz: $x(t) = A\sin(3t) + B\cos(3t)$

$\Rightarrow \dot{x}(t) = 3A\cos(3t) - 3B\sin(3t)$

$\rightarrow 3A\cos(3t) - 3B\sin(3t)$
$+ 7A\sin(3t) + 7B\cos(3t) = \sin(3t) \quad \forall t$

$$\Leftrightarrow \begin{cases} 3A + 7B = 0 \\ 7A - 3B = 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} A = \dfrac{\begin{vmatrix} 0 & 7 \\ 1 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & 7 \\ 7 & -3 \end{vmatrix}} = \dfrac{-7}{-9-49} = \dfrac{7}{58} \\ B = \dfrac{\begin{vmatrix} 3 & 0 \\ 7 & 1 \end{vmatrix}}{-58} = -\dfrac{3}{58} \end{cases}$$

5) $f(x) = \sin(\ln(x)) \qquad f(1) = \sin(0) = 0$

$f'(x) = \cos(\ln(x)) \cdot \frac{1}{x} \qquad f'(1) = \cos(0) \cdot \frac{1}{1} = 1$

$f''(x) = -\sin(\ln(x)) \cdot \frac{1}{x^2} \qquad f''(1) = -0 + 1 \cdot (-1)$
$+ \cos(\ln(x)) \cdot \left(-\frac{1}{x^2}\right) \qquad = -1$

Also $f(1{,}02) \approx \underbrace{0 + 1 \cdot 0{,}02 - 1 \cdot \frac{0{,}02^2}{2}}$

$\underbrace{0{,}02 - 0{,}02 \cdot 0{,}01}_{0{,}0198.}$

6) $Y(s) = \dfrac{1}{s^3+16s} = \dfrac{1}{s} \cdot \dfrac{1}{s^2+16}$

$= \dfrac{A}{s} + \dfrac{Bs + C}{s^2+16}$

$= \dfrac{A(s^2+16) + (Bs+C)s}{s \cdot (s^2+16)}$

$\iff \begin{cases} s^2: & A + B = 0 \\ s^1: & C = 0 \\ s^0: & 16A = 1 \end{cases}$

$\iff A = \frac{1}{16} \wedge B = -\frac{1}{16} \wedge C = 0$

Also $y(t) = \frac{1}{16} - \frac{1}{16}\cos(4t)$.

7)

$\|\vec{a}\| = 2 \quad \mathrm{I}$

$\vec{a} \cdot \vec{b} = 0 \quad \mathrm{II}$

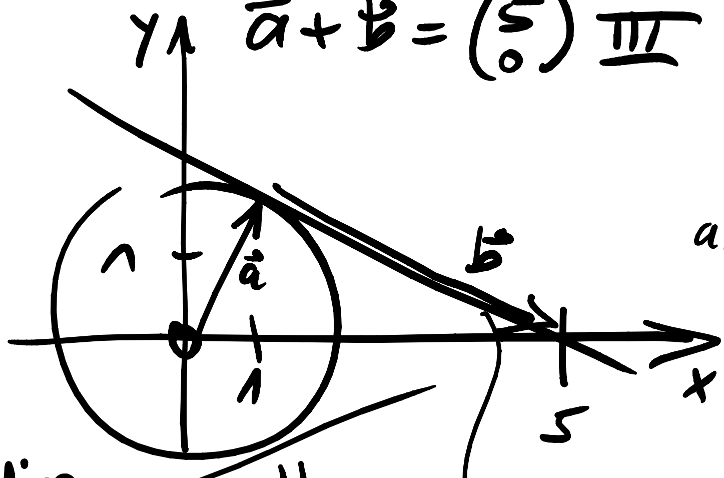
$\vec{a} + \vec{b} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad \mathrm{III}$

$\underbrace{\vec{a}\cdot\vec{a}}_{\overset{\mathrm{I}}{=}4} + \underbrace{\vec{a}\cdot\vec{b}}_{\overset{\mathrm{II}}{=}0} \overset{\mathrm{III}}{=} \vec{a} \cdot \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$\Rightarrow \vec{a} = \begin{pmatrix} 5/4 \\ a_y \end{pmatrix}$

aber $2^2 \overset{\mathrm{I}}{=} \|\vec{a}\|^2 = \frac{25}{16} + a_y^2$

$\Rightarrow a_y = \pm\sqrt{\frac{39}{16}} = \pm\frac{\sqrt{39}}{4}$



((oder diese:))

Geradengl.:

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 5/4 - 5 \\ \sqrt{39}/4 \end{pmatrix}.$

8) $M = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

$$= \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

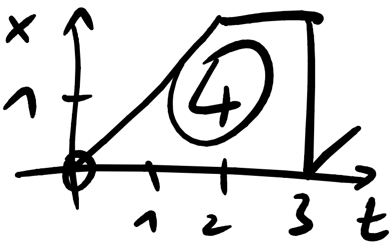
9) $0 \stackrel{!}{=} \begin{vmatrix} 3-\lambda & 7 & 0 \\ 0 & 4-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = (3-\lambda)(4-\lambda)(3-\lambda) + 0 + 0$
$- 0 - 0 - 0$

Also $\lambda = 3 \vee \lambda = 4$.

Die Matrix hat die E.W. 3 und 4.

E.V. dazu: $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ bzw. $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ ((z.B.))

10) $c_0 = \frac{4}{3}$



$$c_3 = \frac{1}{3} \int_0^3 e^{-2\pi i 3 \frac{t}{3}} f(t)\,dt$$

$$= \frac{1}{3} \underbrace{\int_0^2 e^{-2\pi i t} \cdot t\,dt}_{\frac{e^{-2\pi i t}}{-2\pi i} \quad 1} + \underbrace{\frac{1}{3} \int_2^3 e^{-2\pi i t} \cdot 2\,dt}_{\left[\frac{e^{-2\pi i t}}{-2\pi i} \cdot 2\right]_2^3}$$

$\downarrow$

$$\left[\frac{e^{-2\pi i t}}{-2\pi i} \cdot t\right]_0^2 + \frac{1}{2\pi i} \int_0^2 e^{-2\pi i t}\,dt$$

$$= \frac{1}{3}\left(\frac{\cancel{e^{-2\pi i\cdot 2}}^{\,1}}{-\cancel{4}\pi i}\cdot\cancel{4}-0\right)+\frac{1}{6\pi i}\left[\frac{e^{-4\pi i t}}{-2\pi i}\right]_0^2+\frac{1}{3}\left[\frac{e^{-2\pi i t}}{-\cancel{2}\pi i}\cdot\cancel{2}\right]_2^3$$

$$\left(\left(= -\frac{1}{3\pi i}+\underbrace{\xi\,\frac{e^{-2\pi i\cdot 2}-e^{0}}{\xi}}_{0}-\frac{1}{3\pi i}\underbrace{\left(e^{-2\pi i\cdot 3}-e^{-2\pi i\cdot 2}\right)}_{\underbrace{1-1}_{0}}\right.\right.$$

$$\left.\left.= \frac{i}{3\pi}\right)\right)$$

11) z.B. $f(x,y) = y e^{2x} + y^3$

12) $V = \int_0^7 \left(\int_0^{\pi/6}\left(\int_0^{2\pi} 1\cdot d\varphi\right)\sin(\vartheta)\,d\vartheta\right) r^2\,dr$

$$= \frac{7^3}{3}\cdot 2\pi \cdot \underbrace{\int_0^{\pi/6}\sin(\vartheta)\,d\vartheta}_{\underbrace{\left[-\cos(\vartheta)\right]_0^{\pi/6}}_{-\frac{\sqrt{3}}{2}+1}}$$

$$= \frac{7^3}{3}\cdot 2\pi\cdot\left(1-\frac{\sqrt{3}}{2}\right).$$