Mathematik 2

2023-10-06

Musterlösungen

1) $$\underbrace{\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\2\\0\end{pmatrix}}_{1} = \underbrace{\left\|\begin{pmatrix}1\\1\\1\end{pmatrix}\right\|}_{\sqrt{3}}\cdot\underbrace{\left\|\begin{pmatrix}-1\\2\\3\end{pmatrix}\right\|}_{\sqrt{5}}\cdot\cos(\varphi)$$

$$\Rightarrow \varphi = \arccos\left(\frac{1}{\sqrt{15}}\right)$$

2) 2. Spalte ist Doppeltes der 1. Spalte, 3. Spalte ist davon lin. unabh.
Also Spaltenraum = Ebene $\lambda\begin{pmatrix}1\\2\\3\end{pmatrix}+\mu\begin{pmatrix}0\\0\\1\end{pmatrix}$.
Rang = 2.

3)
$$\begin{array}{rrrr|l} 1 & 1 & 0 & 1 & \\ 2 & 0 & -1 & 2 & \cdot(-2) \\ 0 & 3 & 2 & 4 & \\ 1 & 3 & 1 & 5 & \cdot(-1) \end{array}$$

$$\begin{array}{rrrr|l} 1 & 1 & 0 & 1 & \\ 0 & -2 & -1 & 0 & \\ 0 & 3 & 2 & 4 & \cdot\frac{3}{2} \\ 0 & 2 & 1 & 4 & \cdot 1 \end{array}$$

$\downarrow$

$$\begin{matrix} 1 & 1 & 0 & 1 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & \frac{1}{2} & 4 \\ 0 & 0 & 0 & 4 \end{matrix} \quad \lightning$$

Es gibt keine Lösung!

4) • Homogene Form: $y' - 3y = 0$

Ansatz: $y = e^{\lambda x}$

$\Rightarrow \lambda e^{\lambda x} - 3e^{\lambda x} = 0 \Rightarrow \lambda = 3$

Also allg. Lsg. der homog. Form:

$y(x) = A e^{3x}$

• Inhomogene Form: $y' - 3y = e^{2x}$

Ansatz: $y = Be^{2x}$

$\Rightarrow B \cdot 2e^{2x} - 3Be^{2x} = e^{2x}$

$\Rightarrow B \cdot 2 - 3B = 1 \Rightarrow B = -1$

Also spez. Lsg. der inh. Form:

$y(x) = -e^{2x}$

• Allg. Lsg der inh. Form:

$y(x) = Ae^{3x} - e^{2x}$

5) $y' = \frac{\cos(x)}{3y} \Longrightarrow 3y\,dy = \cos(x)\,dx$

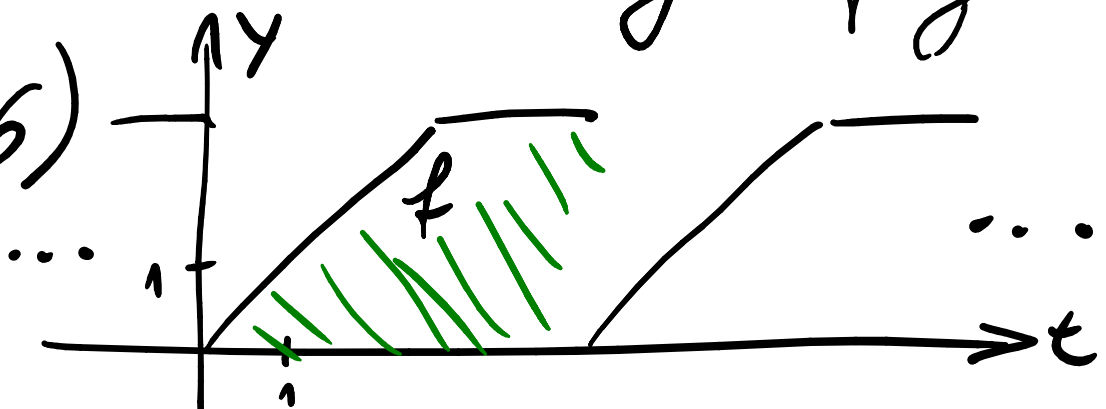
Also $\underbrace{\int_5^{y_1} 3y\,dy}_{\left[\frac{3y^2}{2}\right]_5^{y_1}} = \underbrace{\int_4^{x_1} \cos(x)\,dx}_{\left[\sin(x)\right]_4^{x_1}}$.

$$\Longrightarrow \frac{3y_1^2}{2} - \frac{3\cdot 25}{2} = \sin(x_1) - \sin(4)$$

$$\Longrightarrow y_1 = \pm\sqrt{25 + \frac{2}{3}\left(\sin(x_1) - \sin(4)\right)}$$

+ wegen Anfangsbedingung

6)



abgelesen:

$$c_0 = \frac{10\frac{1}{2}}{5} \left(\left(= 2\frac{1}{10}\right)\right)$$

$$c_2 = \frac{1}{5}\int_0^5 e^{-2\pi i\, 2\frac{t}{5}} f(t)\,dt$$

$$= \frac{1}{5}\left(\int_0^3 e^{-\frac{4}{5}\pi i t}\, t \, dt + \underbrace{\int_3^5 e^{-\frac{4}{5}\pi i t}\cdot 3 \, dt}_{\left[\frac{e^{-\frac{4}{5}\pi i t}}{-\frac{4}{5}\pi i}\cdot 3\right]_3^5}\right)$$

$$e^{-\frac{4}{5}\pi i t} \uparrow \qquad t \downarrow$$

$$\underbrace{\frac{e^{-\frac{4}{5}\pi i t}}{-\frac{4}{5}\pi i} \qquad 1}_{\left[\frac{e^{-\frac{4}{5}\pi i t}}{-\frac{4}{5}\pi i}\cdot t\right]_0^3 - \underbrace{\int_0^3 \frac{e^{-\frac{4}{5}\pi i t}}{-\frac{4}{5}\pi i}\cdot 1 \, dt}_{\left[\frac{e^{-\frac{4}{5}\pi i t}}{\left(-\frac{4}{5}\pi i\right)^2}\right]_0^3}}$$

$$= \frac{1}{\cancel{5}}\left(\frac{e^{-\frac{12}{5}\pi i}}{-\frac{4}{\cancel{5}}\pi i}\cdot 3 - 0 - \frac{e^{-\frac{12}{5}\pi i} - \cancel{e^0}\,1}{-\frac{16}{\cancel{25}}\pi^2} + \frac{\cancel{e^{-4\pi i}}\,1 - e^{-\frac{12}{5}\pi i}}{-\frac{4}{\cancel{5}}\pi i}\cdot 3\right)$$

$$\left(\left(= e^{-\frac{\cancel{12}\,2}{5}\pi i}\left(\cancel{\frac{-3}{4\pi i}} + \frac{1}{\frac{16}{5}\pi^2} \cancel{+ \frac{3}{4\pi i}}\right) - \frac{1}{\frac{16}{5}\pi^2} - \frac{3}{4\pi i}\right)\right)$$

weil ganze Umdrehung

7) Gesuchte Gleichung:

$$2x - y + 3z = c.$$

selber Normalenvektor

$(1|2|1)$ soll drauf sein,

also $c = 2 \cdot 1 - 2 + 3 \cdot 1 = 5.$

8) Prüfe:

$$0 \overset{?}{=} \begin{vmatrix} 1-7 & 4 & 0 & 0 \\ 2 & 3-7 & 0 & 0 \\ 0 & 0 & 4-7 & 0 \\ 0 & 5 & 0 & 1-7 \end{vmatrix} = \begin{vmatrix} -6 & 4 & 0 & 0 \\ 2 & -4 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 5 & 0 & -6 \end{vmatrix}$$

$$= -3 \cdot \begin{vmatrix} -6 & 4 & 0 \\ 2 & -4 & 0 \\ 0 & 5 & -6 \end{vmatrix} = (-3) \cdot (-6) \cdot \underbrace{\begin{vmatrix} -6 & 4 \\ 2 & -4 \end{vmatrix}}_{\neq 0}$$

$\neq 0$, also ist 7 <u>kein</u> E.W.

9) Allg. Lsg.: $y(x) = A e^{3ix} + B e^{-3ix}$

Nehme z.B. DGL $y'' + 9y = 0$.

$$\Downarrow$$

$$\lambda^2 + 9 = 0 \Rightarrow \lambda = \pm 3i$$

10) $sY(s) - 10 + 7Y(s) = 5 \cdot \frac{3}{s^2+9}$

$\Rightarrow Y(s) = \frac{\frac{15}{s^2+9} + 10}{s+7}$

11) $\frac{\partial f}{\partial x} = 1$, $\frac{\partial f}{\partial y} = 2yz^3$, $\frac{\partial f}{\partial z} = 3y^2z^3$

an $(3|2|1)$: $\downarrow$ 1, $\downarrow$ $4 \cdot 1 = 4$, $\downarrow$ $3 \cdot 4 \cdot 1 = 12$

Also $f(3{,}01;\ 2{,}02;\ 0{,}99)$

$\approx 3 + 2^2 \cdot 1^3 + 1 \cdot 0{,}01 + 4 \cdot 0{,}02 - 12 \cdot 0{,}01$

$(\!(= 6{,}97)\!)$

12) $\frac{\partial f}{\partial x} = 2x - 2y \stackrel{!}{=} 0 \Rightarrow x = y$

$\frac{\partial f}{\partial y} = -2x + 8y \stackrel{!}{=} 0 \Rightarrow y = 0$

$\Rightarrow x = 0$

Der Gradient ist nur an $(0|0)$ der Nullvektor (notwendig!).

$$\frac{\partial^2 f}{\partial x^2} = 2, \quad \frac{\partial^2 f}{\partial y^2} = 8, \quad \frac{\partial^2 f}{\partial x \partial y} = -2$$

Also Hesse-Matrix überall gleich $\begin{pmatrix} 2 & -2 \\ -2 & 8 \end{pmatrix}$.

$$\det. = 2 \cdot 8 - (-2)\cdot(-2) = 12 > 0$$

> 0

hinreichend für lok. Min. an $(0|0)$.