Mathematik 1
Klausur vom 2024-01-31
Musterlösungen

1. $\sqrt[3]{2^{x+5}+7} = 10 \iff 2^{x+5}+7 = 1000$

$\iff 2^{x+5} = 993 \iff x+5 = \log_2(993)$

$\iff x = \log_2(993) - 5 \quad \left(\left(= \log_2(993/32)\right)\right)$

2. $\underbrace{z^3+z^2+z}_{z(z^2+z+1)} = 0$

Nebenrechnung:

$z^2+z+1=0$

$\iff z = -\frac{1}{2} \pm \sqrt{\frac{1}{4}-1}$

$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$\Updownarrow$

$z = 0+0i$

$\vee\ z = -\frac{1}{2} + \frac{\sqrt{3}}{2} i \ \vee\ z = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

3. ((Zum Beispiel:))

- $\frac{7}{2} \cdot (x-2)(x-3)$
- $-\frac{7}{2} \cdot (x-2)^2(x-3)$
- $-\frac{7}{200}(x-2)(x-3)(x-101)$

4. Nebenrechnung: $x^2-5x+6=0$

$$\Leftrightarrow x = \frac{5}{2} \pm \sqrt{\frac{25}{4}-6}$$

$$= \frac{5}{2} \pm \frac{1}{2}$$

$$\Leftrightarrow x=2 \lor x=3$$

und $x^2-6x+9=0$

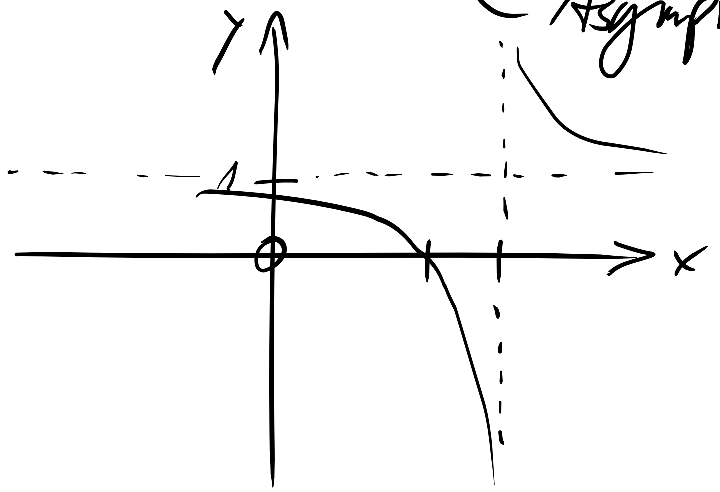
$$\Leftrightarrow x = 3 \pm \sqrt{9-9} = 3$$

Also, $\dfrac{x^2-5x+6}{x^2-6x+9} = \dfrac{(x-2)\cancel{(x-3)}}{(x-3)^{\cancel{2}}}$.

$\Rightarrow$ Nullstelle bei 2, Polstelle bei 3.

$(x-2):(x-3) = 1$ Rest ..

Asymptote für $x \to \pm\infty$



5. $$\frac{d}{dx}\left(\frac{e^{2x}}{x^4+1}\right)^3 = 3\left(\frac{e^{2x}}{x^4+1}\right)^2 \cdot \frac{e^{2x}\cdot 2\cdot(x^4+1) - e^{2x}\cdot 4x^3}{(x^4+1)^2}$$

$$6.\quad V = \pi\int_0^{\pi/2} \sqrt{x\cos(x)}^{\,2}\,dx = \pi\int_0^{\pi/2} x\cos(x)\,dx$$

sind beide für $0 \le x \le \frac{\pi}{2}$ nicht negativ

$$= \pi\int_0^{\pi/2} \underset{\substack{\downarrow\\ 1}}{x}\;\underset{\substack{\uparrow\\ \sin(x)}}{\cos}(x)\,dx = \pi\left(\underbrace{\Big[x\sin(x)\Big]_0^{\pi/2}}_{\frac{\pi}{2}1-0\cdot 0} - \underbrace{\int_0^{\pi/2} 1\cdot\sin(x)\,dx}_{\big[-\cos(x)\big]_0^{\pi/2}}\right)$$

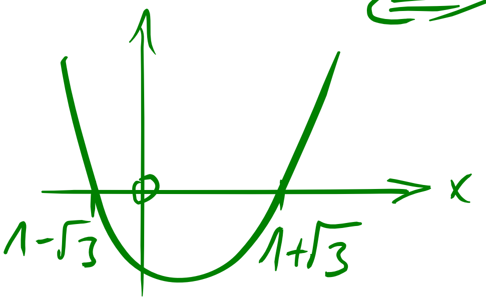
$$= \pi\left(\frac{\pi}{2} + \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} - \underbrace{\cos(0)}_{1}\right) = \frac{\pi^2}{2} - \pi.$$

7. $|x-1|^2 \geq 3 \iff (x-1)^2 \geq 3$

Betrag egal wegen Quadrat!

$\iff x^2 - 2x \underbrace{+1-3}_{-2} \geq 0$

Nebenrechnung: $x^2-2x-2=0$
$\iff x = 1 \pm \sqrt{1+2}$
$= 1 \pm \sqrt{3}$

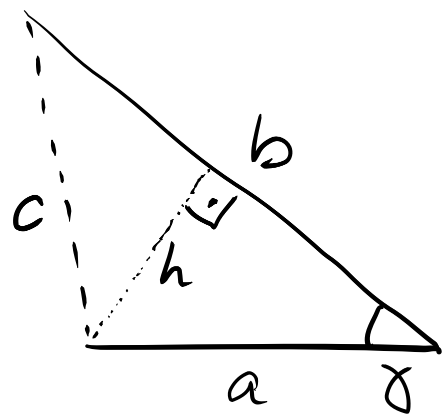


$\iff x \leq 1-\sqrt{3} \lor x \geq 1+\sqrt{3}$

Also $\mathbb{L} = (-\infty; 1-\sqrt{3}] \cup [1+\sqrt{3}; \infty)$.

8.

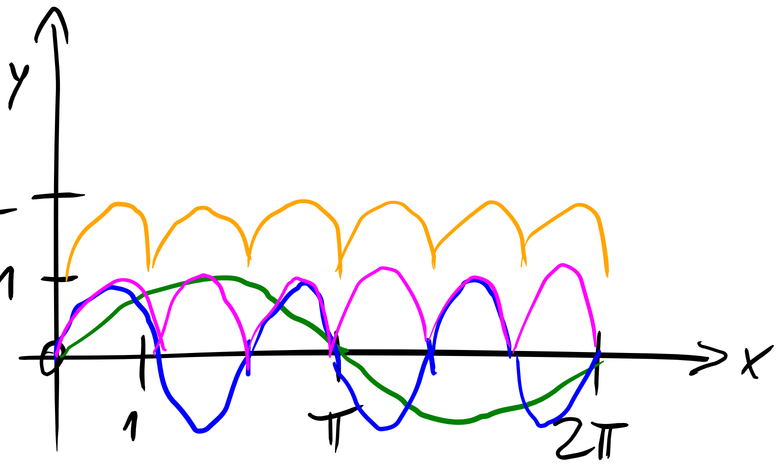
Die Lage von c und damit der Flächeninhalt sind eindeutig bestimmt!



$h = a \cdot \sin(\gamma)$

$A = \frac{1}{2} h \cdot b = \frac{1}{2} \cdot 3 \cdot \sin(40°) \cdot 5$

9.



$y = \cos\left(x - \frac{\pi}{2}\right)$
$= \sin(x)$

$y = \cos\left(3x - \frac{\pi}{2}\right)$

$y = \left|\cos\left(3x - \frac{\pi}{2}\right)\right|$

$y = \left|\cos\left(3x - \frac{\pi}{2}\right)\right| + 1$

10.
$$\frac{\cos(n^8) + \sqrt[3]{n^6+7}}{e^{8-n} + 9n^2} = \frac{\underbrace{\frac{1}{n^2}\cos(n^8)}_{0} + \frac{\sqrt[3]{n^6+7}}{n^2}}{\underbrace{\frac{e^{8-n}}{n^2}}_{0} + 9\frac{\cancel{n^2}}{\cancel{n^2}}}$$

$$= \sqrt[3]{\underbrace{\frac{\cancel{n^6}}{\cancel{n^6}}}_{1} - \frac{7}{n^6}} \longrightarrow \sqrt[3]{1} = 1$$

Also $\cdot/\cdot \longrightarrow \frac{1}{9}$.

11. $$\int_4^5 \frac{\sin(\ln(x))}{x}\,dx$$

Substitution: $u = \ln(x)$

$$\Rightarrow \frac{du}{dx} = \frac{1}{x}$$

$$\Rightarrow du = \frac{1}{x}\,dx$$

$$= \int_{\ln(4)}^{\ln(5)} \sin(u)\,du = \Big[-\cos(u)\Big]_{\ln(4)}^{\ln(5)}$$

$$= -\cos(\ln(5)) + \cos(\ln(4))$$

12. $$2 = \sigma = \sqrt{\underbrace{E[X^2]}_{0{,}9\cdot 1^2 + 0{,}1a^2} - \underbrace{E[X]}_{(0{,}9\cdot 1 + 0{,}1a)}{}^2}$$

$$\Rightarrow 4 = 0{,}9 + 0{,}1a^2 - \underbrace{0{,}9^2}_{0{,}81} - \underbrace{2\cdot 0{,}9\cdot 0{,}1a}_{0{,}18} - \underbrace{0{,}1^2}_{0{,}01}a^2$$

$$\Rightarrow 0 = 0{,}09a^2 - 0{,}18a - 3{,}91$$

$$\Rightarrow 0 = a^2 - 2a - \frac{3{,}91}{0{,}09}$$

$$\Rightarrow a = 1 \pm \sqrt{1^2 + \frac{3{,}91}{0{,}09}} \quad \left(\left(= 1 + \sqrt{\frac{4{,}00}{0{,}09}} = 1 + \frac{20}{3} = \frac{23}{3} \right)\right)$$

$\oplus$, weil $a > 1$ sein soll.