Wind- und Wasserkraft

B. Eng. Regenerative Energien / Elektrotechnik
Klausur vom 27. Januar 2025: Musterlösungen

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Lösungen von DeepSeek R1, nach erneuter Bitte, die komplett in LaTeXzu formatieren; Anmerkungen und Korrekturen in Rot von Jörn Loviscach

Gedanken

Okay, let's tackle these problems one by one. I need to make sure I understand each question and apply the right concepts. Let's start with the first one about the Tiefdruckgebiet on the Southern Hemisphere. Hmm, Tiefdruckgebiete rotate clockwise in the Southern Hemisphere due to the Coriolis effect. The three main forces are pressure gradient force, Coriolis force, and centrifugal force. I should draw the forces at a point: pressure gradient towards the center, Coriolis to the left of the wind direction (which would be outward here?), and centrifugal force outward. Wait, need to check directions again.

Next, reducing noise from wind turbines. Common methods include serrated trailing edges on blades, optimizing blade design to reduce turbulence, using sound dampening materials, and increasing the distance to residential areas. Maybe three measures like those.

For the shadow of a wind turbine at 23°N on June 21st. June 21 is the summer solstice in the Northern Hemisphere, so the sun is directly over 23.5°N. Since the turbine is at 23°N, the sun will be slightly north, so the shadow points south. The length depends on the solar altitude. At solar noon, the solar altitude is 90° - (23.5 - 23) = 89.5°, almost overhead. So the shadow would be very short, almost directly south.

The pumped storage question: energy stored is gravitational potential energy. E = mgh. The volume is area times depth (50m), and mass is volume times density (1000 kg/m^3). So E = A * 50 * 1000 * 9.81 * 300. Solve for A. Wait, but the energy is

10 GWh, which needs to be converted to joules. 10 GWh = 10e9 * 3600 = 3.6e13 J. So A = E / (50*1000*9.81*300). Let me check units.

Pressure in the pipeline 500m below the reservoir. Using hydrostatic pressure: P = ρgh. The flow velocity might not be needed if they mention some info is extra. So just calculate ρgh. 1000 kg/m^3 * 9.81 m/s^2 * 500 m = 4,905,000 Pa or 4905 kPa.

Volllaststunden and capacity factor with 36.5 days of calm. Total hours in a year: 8760. Hours without wind: 36.5*24=876. So operating hours: 8760-876=7884. Volllaststunden is equivalent to energy produced divided by rated power. But capacity factor is actual energy / (rated power * 8760). If the turbine can't operate at all during calm, then the capacity factor would be 7884/8760 $\approx$ 0.9, but that seems too high. Wait, maybe the question is about the number of full load hours being less, but I need to think. If there are 36.5 days with no wind, the turbine operates 365-36.5=328.5 days. So 328.5 *24=7884 hours. But Volllaststunden is the equivalent hours at full power to produce the same energy. If the turbine sometimes operates below capacity, the Volllaststunden would be less than actual operating hours. But without specific data, maybe the capacity factor is (7884/8760) $\approx$ 90%, but actually, the capacity factor is typically around 30-50% for wind. Hmm, perhaps the answer is that the number of full load hours is (annual energy)/(rated power), and if there's 36.5 days of no generation, then it's reduced by that time. But the question states "Windstille herrscht", so no generation during those days. So the remaining days have variable generation. The capacity factor would depend on the actual production. However, the question is asking what can be said: the number of full load hours would be less than (365-36.5)*24, but without knowing the actual production, maybe it's just that the capacity factor is lower because of the downtime. Wait, the question is a bit unclear. Maybe just state that the number of full load hours is (total energy)/(rated power), and since 10% of the year has no wind, it reduces the possible maximum. But I need to answer precisely. Maybe the capacity factor is (1 - 36.5/365) times the capacity factor without downtime, but not sure. Need to think again.

Ultraleichtflugzeug: Gleitzahl 25, mass 500 kg, speed 100 km/h. Required power to stay aloft. The glide ratio is lift over drag. Power needed is drag times velocity. Lift equals weight: L = mg = 500*9.81=4905 N. Drag D = L / Gleitzahl = 4905 /25=196.2 N. Power P = D * v. Convert 100 km/h to m/s: 100/3.6 $\approx$27.78 m/s. So P = 196.2 *27.78 $\approx$ 5450 W or 5.45 kW.

Betz's law: if wind is slowed to 3/4v before rotor and 1/2v after. The power coefficient is (16/27) Betz limit. But here the velocities are different. The formula is Cp = 4a(1-a)^2, where a is the axial induction factor. Wait, the wind speed at rotor is average of upstream and downstream? If upstream is v, at rotor it's 3/4v, downstream 1/2v. Then a = (v - 3/4v)/v = 1/4. Then Cp = 4*(1/4)*(1 - 1/4)^2 = 4*(0.25)*(0.75)^2 = 4*0.25*0.5625 = 0.5625. But Betz limit is 16/27$\approx$0.5926. So Cp here is 0.5625, which is below Betz. So the Leistungsbeiwert is 0.5625 or 56.25

Wind profile exponent: Using the power law v2/v1 = (z2/z1)^α. Given v1=10 m/s at 100m, v2=12 m/s at 200m. So 12/10 = (200/100)^α $\rightarrow$ 1.2 = 2^α. Take ln: ln(1.2) = α ln(2) $\rightarrow$ α = ln(1.2)/ln(2) $\approx$0.1823/0.6931$\approx$0.263.

Windstatistiken: Standorte A and B have the same mean wind speed. A has a broader distribution (Weibull with lower shape parameter k=2), B is more peaked (k=5). For energy yield, a site with higher frequency of rated wind speeds is better. Site B has higher k, meaning more consistent winds around the mean. But wind power is proportional to v^3, so higher frequency of higher winds is better. However, A has a longer tail, which might contribute more to energy. Wait, but if A has lower k (Weibull shape parameter), it has a wider spread with more frequent low and high winds. Since power is v^3, the higher wind speeds contribute disproportionately. So site A might have higher energy production. For the grid, site B is better because more consistent, less fluctuation. So A is better for ertrag, B better for grid.

Schalldruckpegel: Adding sources. 39 dB each. Want total 45 dB. The formula is Lp_total = 10*log10(n) + Lp_single. So 45 = 10 log(n) +39 $\rightarrow$ 6 =10 log(n) $\rightarrow$ log(n)=0.6 $\rightarrow$ n=10^0.6≈3.98. So 4 turbines. Since you can't have a fraction, 4. But check: 4 sources would be 39 +10*log10(4)=39+6.02≈45.02 dB. So 4 turbines.

Generator and grid: Voltage drop in the line. The generator delivers P=1 MW. The line has R=1Ω, L=10mH, X=2π*50*0.01=3.14Ω. The impedance Z=$\sqrt{}$(R^2+X^2)=$\sqrt{}$(1+9.86)=$\sqrt{}$10.86≈3.296Ω. The voltage at the generator Vg and grid Vn=10kV. We need |Vg| = |Vn|. Using the power transfer equation: P = (Vg Vn / Z) * cos(φ - θ) - (Vn^2 / Z) cosθ, but this might be complex. Alternatively, using the approximation for small voltage differences: Vg ≈ Vn + I*(R +jX). The power is P = Vn * I * cosφ. So I = P/(Vn cosφ). The voltage drop is I*(R +jX). The magnitude of Vg should equal Vn. So |Vn + I*(R +jX)| = Vn. Let's square both sides: (Vn + I R)^2 + (I X)^2 = Vn^2. Expand: Vn^2 + 2 Vn I R + I^2 R^2 + I^2 X^2 = Vn^2 $\rightarrow$ 2 Vn I R + I^2 (R^2 + X^2) =0. Divide by I (assuming I≠0): 2 Vn R + I (R^2 + X^2)=0 $\rightarrow$ I= -2 Vn R / (R^2 + X^2). But I is positive, so this seems conflicting. Maybe another approach. Let me use phasors. Let Vg = Vn + ΔV, where ΔV = I*(R +jX). The magnitude |Vg| = |Vn + ΔV| = Vn. So the phasor ΔV must be perpendicular to Vn. Therefore, the angle of ΔV is 90 degrees relative to Vn. So the current I must have a phase such that the voltage drop is perpendicular. Since ΔV = I*(R +jX), and Vn is the reference (0 angle), then ΔV must be a phasor that is perpendicular to Vn, i.e., purely imaginary. So the real part of ΔV must be zero. Therefore, I*(R) must be zero, which implies R=0, but here R=1Ω. Hmm, maybe not. Alternatively, using the condition that the voltage magnitudes are equal, so the phasor diagram forms an isosceles triangle. Let's denote Vg = Vn + I*(R +jX). The magnitude squared: Vn^2 = (Vn + I R)^2 + (I X)^2. Expanding: Vn^2 = Vn^2 + 2 Vn I R + I^2 R^2 + I^2 X^2 $\rightarrow$ 0 = 2 Vn I R + I^2 (R^2 + X^2). Solving for I: I = -2 Vn R / (R^2 + X^2). But I is positive, so this suggests that R must be negative, which it's not. Therefore, this implies no solution unless R=0, which contradicts. Therefore, maybe the only solution is when the reactive power compensates the voltage drop. Alternatively, using the fact that the voltage drop is mainly reactive. Let's consider that for small voltage drops, the angle Δ is small, so Vg ≈ Vn + (P R + Q X)/Vn. To have |Vg|=|Vn|, the real part should cancel. So (P R + Q X)/Vn =0 $\rightarrow$ P R + Q X =0 $\rightarrow$ Q = - P R /X. The power factor is cosφ = P / S, where S=$\sqrt{}$(P^2 + Q^2). So Q= -P R/X. Then S= P $\sqrt{}$(1 + $(R/X)^2$). So cosφ = 1 / $\sqrt{}$(1 + $(R/X)^2$). Plugging R=1, X≈3.14: cosφ=1/$\sqrt{}$(1 + (1/3.14)^2)≈1/$\sqrt{}$(1 +0.101)≈1/1.049≈0.953. Then current I= S / Vn = (P / cosφ)/ Vn = (1e6 /0.953)/10e3 ≈ 1e6/(0.953*10e3)≈105 A. Wait, but need to check. Alternatively, from Q= -P R /X = -1e6 *1 /3.14 ≈ -318471 VAR. Then S=$\sqrt{}$($1e6^2$ +318471^2)≈1,048,800 VA. cosφ=

P/S =1e6 /1,048,800≈0.953. Current I= S/Vn=1,048,800 /10e3≈104.88 A. So the answer is cosφ≈0.953 and I≈104.9 A.

Lösungen

1. **Tiefdruckgebiet auf der Südhalbkugel:** Die Luft zirkuliert **im Uhrzeigersinn** um das Tiefdruckgebiet. Die drei Kräfte an einer Stelle:

   - **Druckgradientenkraft** (rot): Zum Zentrum hin,
   - **Corioliskraft** (blau): Nach links der Strömungsrichtung (nach außen),
   - **Zentrifugalkraft** (grün): Von der Drehbewegung weg (nach außen).

nicht so wichtig

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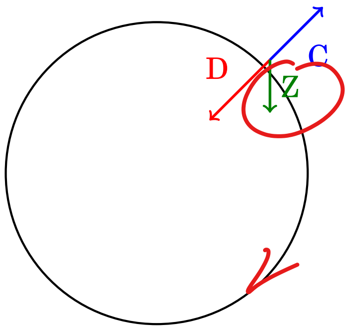

Im Uhrzeigersinn rotierende Windströmung

2. **Geräuschreduktion bei Windenergieanlagen:** Maßnahmen umfassen:

   - Sägezahnförmige Blattspitzen („Serrations“) zur Reduktion von Wirbelschleppen,
   - Optimierte Blattgeometrie und -oberfläche zur Minimierung von Turbulenzen,
   - Aktive Drehzahlregelung bei niedrigen Windgeschwindigkeiten.

na ja

3. **Schattenlänge am 21. Juni bei 23°N:** Am 21. Juni steht die Sonne nahe 23,5°N (nördlicher Wendekreis). Da die Turbine bei 23°N steht, ist der Schatten **nach Süden gerichtet** und extrem kurz:

warum Betrag ?

$$\text{Länge} \approx \frac{\text{Höhe}}{\tan(90° - |23{,}5° - 23°|)} \approx \boxed{\text{vernachlässigbar kurz}}$$

4. **Fläche des Oberbeckens:** Energie $E = \rho \cdot g \cdot h \cdot V$ mit $V = A \cdot T$:

Einheiten!

f

$$A = \frac{E}{\rho \cdot g \cdot h \cdot T} = \frac{10 \cdot 10^9 \cdot 3600}{1000 \cdot 9{,}81 \cdot 300 \cdot 50} \approx \boxed{244\,200\,\mathrm{m}^2}$$

5. **Druck in der Leitung:** Hydrostatischer Druck dominiert:

$$P = \rho \cdot g \cdot h = 1000 \cdot 9{,}81 \cdot 500 = \boxed{4{,}905\,\mathrm{MPa}}$$

+ 1 bar

Hinweis: Strömungsgeschwindigkeit ist irrelevant.

6. **Volllaststunden und Kapazitätsfaktor:** Volllaststunden: $365 - 36{,}5 = 328{,}5\,\text{Tage} \times 24 = \boxed{7884\,\text{h}}$. Kapazitätsfaktor: $\frac{7884}{8760} \approx \boxed{0{,}9}$ (theoretisch; real geringer durch Teillast).

betonen!

7. **Leistung des Ultraleichtflugzeugs:** Gleitzahl $= \frac{L}{D} = 25$, $D = \frac{mg}{25}$:

$$P = D \cdot v = \frac{500 \cdot 9{,}81}{25} \cdot \frac{100}{3{,}6} \approx \boxed{5{,}45\,\text{kW}}$$

Einheit!

8. **Leistungsbeiwert nach Betz:** Mit $v_{\text{vor}} = \frac{3}{4}v$, $v_{\text{nach}} = \frac{1}{2}v$:

$$C_P = \frac{16}{27} \cdot \left(1 - \left(\frac{1/2}{3/4}\right)^3\right) = \boxed{0{,}5625} \quad (56{,}25\,\%)$$

f

✓ ??!?

9. **Exponent im Windprofil:** Potenzgesetz: $\frac{v_2}{v_1} = \left(\frac{z_2}{z_1}\right)^{\alpha}$:

$$\alpha = \frac{\ln(12/10)}{\ln(200/100)} = \frac{\ln(1{,}2)}{\ln(2)} \approx \boxed{0{,}263}$$

10. **Windstatistiken Standorte A und B: Ertrag:** Standort A (breitere Verteilung $\rightarrow$ höhere v^3-Beiträge). **Stromnetz:** Standort B (engere Verteilung $\rightarrow$ gleichmäßigere Einspeisung). Begründung: Turbulenzen in A führen zu starken Leistungsschwankungen.

11. **Schalldruckpegel von Windturbinen:** Pegelzunahme: $45 = 39 + 10 \cdot \log_{10}(n) \Rightarrow n = 10^{0{,}6} \approx \boxed{4}$.

12. **Blindleistungskompensation:** Für $|V_{\text{Gen}}| = |V_{\text{Netz}}|$:

$$\cos(\phi) = \frac{X}{\sqrt{R^2 + X^2}} = \frac{3{,}14}{\sqrt{1^2 + 3{,}14^2}} \approx \boxed{0{,}953}$$

Stromstärke:

$$I = \frac{P}{V \cdot \cos(\phi)} = \frac{1 \cdot 10^6}{10 \cdot 10^3 \cdot 0{,}953} \approx \boxed{105\,\text{A}}$$