Designing No-Surprise Teapots

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1 Introduction

Many designs of teapots exhibit bad pouring behaviors: Depending on the amount of tea present in the pot, it may merely drip or may suddenly spourt out. Such effects are common for teapots with concavities, but are not limited to this class. We introduce a physical quantity called Tea Flow Rate (TFR) to describe such irregularities. We created design software that interactively displays the TFR characteristics of a 3D teapot model, allowing the user to apply changes to the model and immediately see the effect they have.

2 Method

To define the TFR, you have to imagine that a full teapot is slowly tilted to empty it completely. Let $V(\phi)$ be the volume remaining inside when the angle ϕ is reached. Then we define $\mathrm{TFR}(\phi) = -dV(\phi)/d\phi$, that is, the TFR measures the volume poured per infinitesimal angle. The TFR can be computed through an area integral, see Figure 1: $\mathrm{TFR}(\phi) = \iint_A r\,dA$, where A is the tea-level surface at the current angle, r denotes the distance to the axis through the spout's highest point (which depends on ϕ), and dA is the area element. Stokes' theorem turns this 2D integral into a line integral.

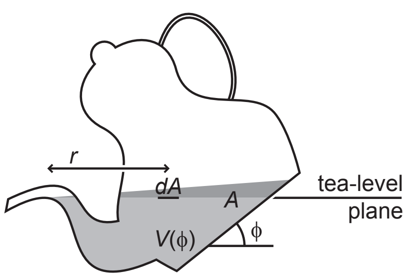

Figure 1: The Tea Flow Rate is defined as the derivative of the accumulated volume poured with respect to the angle of tilt. The darker shading highlights the outflowing infinitesimal volume.

Our software (see Figure 2) allows the user to specify a profile curve. A closed surface of revolution serving as the body of the teapot is generated from the profile. Two further curves are used as extrusion paths to form the spout and the handle; the spout can optionally be tapered. All three curves are editable Bézier curves. A bitmap image may serve as a guide. As the user edits the spout's spine and the body's profile, the software updates the TFR diagram so that low-flow regions and sudden jumps become apparent. The user may also click into the TFR diagram. Then the software rotates the teapot to the corresponding angle ϕ; the current tea-level plane is displayed in the 3D view as a semitransparent rectangle.

To compute the TFR and display the teapot in 3D, its body is tessellated into triangles. The tea-level plane intersects the spout at its highest point, which depends on ϕ. We compute this plane and intersect it with the body's collection of triangles to find a set of line segments. These may form a closed polygon or—for complex shapes—a union of several closed polygons. The line integral for the TFR reduces to a sum over all line segments: $\mathrm{TFR}(\phi) = \sum_i (y_{i2} - y_{i1})(2(x_{i1}y_{i1} + x_{i2}y_{i2}) + x_{i1}y_{i2} + x_{i2}y_{i1})/3$, where x and y refer to a Cartesian coordinate system on the tea-level plane. The origin of this system is located at the highest point of the spout; its x axis points perpendicular to the drawing plane of Figure 1; and y takes the role of r. The indices 1 and 2 denote the start and end points, respectively, of a line segment. Every line segment inherits such a start/end order from the normals of the surface's triangles. Thus, there is no need to sort the segments to arrange them along the intersection curve.

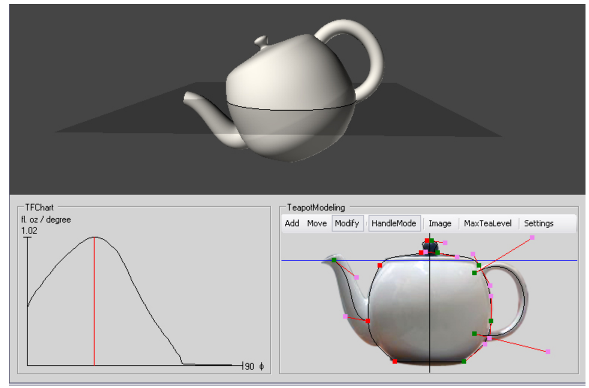

Figure 2: The software prototype offers a 3D preview, an interactive TFR display, and a 2D geometry editor.

The user can define the teapot's actual height to see the TFR results in real-world units. The solution also supports partially filled teapots: The user may set the initial tea-level, from which the software computes the initial volume V_0. Tea starts to flow out at the angle ϕ_0 with $V_\mathrm{T} - V_0 = \int_0^{\phi_0} \mathrm{TFR}(\phi)\,d\phi$, where V_T is the total volume of the body. Thus, the angle ϕ_0 can be computed from the shape; the TFR curve display is set to zero for smaller angles.

3 Results and Outlook

Our tool lets users easily identify and improve bad teapot designs. The solution is based on several approximations: First, it neglects the tea volume stored inside the spout. This is reasonable since a typical teapot's spout stores only $1/100$ of the total volume. Second, we do not take dynamic effects into account, such as a narrow spout blocking the tea flow or the time it takes the tea to reach the spout. Thus, the results are only valid for cautious pouring. Finally, we assume that the tea-level is the same throughout the teapot. This is not valid for sophisticated designs containing zigzag shapes where tea can be trapped above the general tea-level. A future version of our software may address these restrictions. At any rate, the generalization to coffeepots is obvious. However, since even moderate doses of caffeine lead to shakiness, an analysis of coffeepots may require taking the mentioned dynamic phenomena into account.

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